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A NOTE ON H^p_w-BOUNDEDNESS OF RIESZ TRANSFORMS AND θ-CALDERÓN-ZYGMUND OPERATORS THROUGH MOLECULAR CHARACTERIZATION

Luong Dang Ky

(*University of Orleans, France)*

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Abstract. Let $0 < p \leq 1$ and w in the Muckenhoupt class A_1. Recently, by using the weighted atomic decomposition and molecular characterization, Lee, Lin and Yang[11] established that the Riesz transforms $R_j, j = 1, 2, \cdots, n$, are bounded on $H^p_w(\mathbf{R}^n)$. In this note we extend this to the general case of weight w in the Muckenhoupt class A_∞ through molecular characterization. One difficulty, which has not been taken care in [11], consists in passing from atoms to all functions in $H^p_w(\mathbf{R}^n)$. Furthermore, the H^p_w-boundedness of θ-Calderón-Zygmund operators are also given through molecular characterization and atomic decomposition.

Key words: *Muckenhoupt weight, Riesz transform, Calderón-Zygmund operator*

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1 Introduction and Preliminaries

Calderón-Zygmund operators and their generalizations on Euclidean space $\mathbf{R}^n$ have been extensively studied, see for example[7,14,18,15]. In particular, Yabuta[18] introduced certain θ-Calderón-Zygmund operators to facilitate his study of certain classes of pseudo-differential operator.

Definition 1.1. *Let θ be a nonnegative nondecreasing function on $(0,\infty)$ satisfying*

$$\int_0^1 \frac{\theta(t)}{t}\mathrm{d}t < \infty.$$

A continuous function $K : \mathbf{R}^n \times \mathbf{R}^n \setminus \{(x,x) : x \in \mathbf{R}^n\} \to \mathbf{C}$ is said to be a θ-Calderón-Zygmund *singular integral kernel if there exists a constant $C > 0$ such that*

$$|K(x,y)| \leq \frac{C}{|x-y|^n}$$

for all $x \neq y$,

$$|K(x,y) - K(x',y)| + |K(y,x) - K(y,x')| \leq C\frac{1}{|x-y|^n}\theta\Big(\frac{|x-x'|}{|x-y|}\Big)$$

for all $2|x-x'| \leq |x-y|$.

A linear operator $T : \mathcal{S}(\mathbf{R}^n) \to \mathcal{S}'(\mathbf{R}^n)$ is said to be a θ-Calderón-Zygmund operator if T can be extended to a bounded operator on $L^2(\mathbf{R}^n)$ and there exists a θ-Calderon-Zygmund singular integral kernel K such that for all $f \in C_c^\infty(\mathbf{R}^n)$ and all $x \notin$supp f, we have

$$Tf(x) = \int_{\mathbf{R}^n} K(x,y)f(y)\mathrm{d}y.$$

When

$$K_j(x,y) = \pi^{-(n+1)/2}\Gamma\Big(\frac{n+1}{2}\Big)\frac{x_j-y_j}{|x-y|^{n+1}}, \qquad j = 1,2,\cdots,n,$$

then they are the classical Riesz transforms denoted by R_j.

It is well-known that the Riesz transforms $R_j, j = 1,2,\cdots,n$, are bounded on unweighted Hardy spaces $H^p(\mathbf{R}^n)$. There are many different approaches to prove this classical result (see [11, 9]). Recently, by using the weighted molecular theory (see [10]) and combined with García-Cuerva's atomic decomposition [5] for weighted Hardy spaces $H^p_w(\mathbf{R}^n)$, the authors in [11] established that the Riesz transforms $R_j, j = 1,2,\cdots,n$, are bounded on $H^p_w(\mathbf{R}^n)$. More precisely, they proved that $\|R_jf\|_{H^p_w} \leq C$ for every w-$(p,\infty,ts-1)$-atom where $s,t \in \mathbf{N}$ satisfy $n/(n+s) < p \leq n/(n+s-1)$ and $((s-1)r_w+n)/(s(r_w-1))$ with r_w is the *critical index of w for the reverse Hölder condition.* Remark that this leaves a gap in the proof. Similar gaps exist in some litteratures, for instance in [10, 15] when the authors establish H^p_w-boundedness of Calderón-Zygmund type operators. Indee d, it is now well-known that (see [1]) the argument *"the operator T is uniformly bounded in $H^p_w(\mathbf{R}^n)$ on w-(p,∞,r)-atoms, and hence it extends to a bounded operator on $H^p_w(\mathbf{R}^n)$"* is wrong in general. However, Meda, Sjögren and Vallarino [13] establishes that (in the setting of unweighted Hardy spaces) this is correct if one replaces L^∞-atoms by L^q-atoms with $1 < q < \infty$. Later, the authors in [2] extended these results to the weighted anisotropic Hardy spaces. More precisely, it is claimed in [2] that the operator T can be extended to a bounded operator on $H^p_w(\mathbf{R}^n)$ if it is uniformly bounded on w-(p,q,r)-atoms for $q_w < q < \infty, r \geq [n(q_w/p-1)]$ where q_w is the *critical index of w.*