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BMO BOUNDEDNESS FOR BANACH SPACE VALUED SINGULAR INTEGRALS

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Abstract. In this paper, we consider a class of Banach space valued singular integrals. The L^p boundedness of these operators has already been obtained. We shall discuss their boundedness from BMO to BMO. As applications, we get BMO boundedness for the classic g-function and the Marcinkiewicz integral. Some known results are improved.

Key words: BMO, *Banach space valued singular integral*

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1 Introduction

Let H be a Banach space. We denote by $L^p_H, 1 \le p \le +\infty$ the space of H-valued strongly measurable functions g on $\mathbf{R}^n$ such that

$$\|g\|_{L^p_H} = \left(\int_{\mathbf{R}^n} \|g\|^p_H \mathrm{d}x\right)^{1/p} < +\infty, \quad 1 \le p < \infty$$

and when $p = \infty$,

$$\|g\|_{L^\infty_H} = \mathrm{ess\ sup}\|g\|_H < +\infty.$$

The corresponding sharp function is defined as

$$g^\sharp(x) = \sup_{x\in B} \frac{1}{|B|}\int_B \|g(y) - g_B\|_H \mathrm{d}y,$$

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where B denotes any ball in $\mathbf{R}^n$ and g_B is the average of g over B. Finally we define BMO(H) to be the space of all H-valued locally integrable functions g such that

$$\|g\|_{\mathrm{BMO}(H)} = \|g^\sharp\|_{L^\infty(\mathbf{R}^n)} < +\infty.$$

Now we introduce the concept of H-valued singular integral. Let $K(x)$ be an H-valued strongly measurable function defined on $\mathbf{R}^n\backslash\{0\}$, which is also locally integrable in this domain. As we shall take $K(x)$ as the kernel of singular integrals, we present the following continuity requirements which are first introduced by Rubio de Francia, Ruiz and Torrea in [8].

Given $1 \le r \le +\infty$, we call K satisfies the condition D_r if there is a sequence $\{c_k\}_{k=1}^\infty \in l^1$ such that for all $k \ge 1$ and $y \in \mathbf{R}^n$,

$$\left(\int_{S_k(y)} \|K(x-y)-K(x)\|_H^r \mathrm{d}x\right)^{1/r} \le c_k|S_k(y)|^{\frac{1}{r}-1}.$$

Here $S_k(y)$ denotes the spherical shell $\{x\in\mathbf{R}^n:\ 2^k|y| \le |x| \le 2^{k+1}|y|\}$. It is not hard to check that if

$$\|K(x-y)-K(x)\|_H \le C\frac{|y|}{|x|^{n+1}}, \quad |x|>2|y|,$$

then K satisfies D_∞. And D_1 condition is equivalent to the familiar Hömander's condition

$$\int_{|x|>2|y|} \|K(x-y)-K(x)\|_H \mathrm{d}x < +\infty.$$

Besides, D_{r_1} implies D_{r_2} if $r_1 > r_2$. Finally, we call a linear operator T mapping functions into H-valued functions a singular integral operator if

(i) T is bounded from $L^2(\mathbf{R}^n)$ to $L^2_H(\mathbf{R}^n)$;

(ii) There exists a kernel K satisfying D_1 such that

$$Tf(x) = \int_{\mathbf{R}^n} K(x-y)f(y)\mathrm{d}y$$

for every compactly supported f and a.e. $x \notin \mathrm{supp}(f)$.

In [8], the authors proved that such operator can be extended to bounded operators on all $L^p(\mathbf{R}^n), 1<p<\infty$ and satisfy

(a) $\|Tf\|_{L^p_H(\mathbf{R}^n)} \le C\|f\|_{L^p(\mathbf{R}^n)},\ 1<p<\infty$;

(b) $\|Tf\|_{L^1_H} \le C\|f\|_{H^1}$;

(c) $\|Tf\|_{\mathrm{BMO}(H)} \le C\|f\|_{L^\infty},\ f\in L^\infty_c(\mathbf{R}^n)$.

The aim of this paper is to obtain BMO to BMO boundedness for such singular operator. If T is the usual scalar valued singular integral, then it in fact already maps BMO to BMO with