Anal. Theory Appl.
Vol. 27, No. 4 (2011), 365–376
DOI10.1007/s10049-011-0365-z

BOUNDEDNESS OF COMMUTATORS FOR MARCINKIEWICZ INTEGRALS ON WEIGHTED HERZ-TYPE HARDY SPACES

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Received Dec. 2, 2010

Abstract. In this paper, the authors study the boundedness of the operator μ_Ω^b, the commutator generated by a function $b \in \text{Lip}_\beta(\mathbf{R^n})(0<\beta<1)$ and the Marcinkiewicz integral μ_Ω on weighted Herz-type Hardy spaces.

Key words: *Marcinkiewicz integral, commutator, weighted Herz space, Hardy space*

AMS (2010) subject classification: 42B20, 42B25

1 Introduction and Main Result

Let S^{n-1} denote the unit sphere of $\mathbf{R^n}(n\geq 2)$ with Lebesgue measure $d\sigma = d\sigma(x')$. Let $\Omega\in L^1(S^{n-1})$ be homogeneous of degree zero on $\mathbf{R^n}$ and satisfy the cancelation condition

$$\int_{S^{n-1}}\Omega(x')\mathrm{d}\sigma(x')=0,$$

where $x'=x/|x|$ for any $x\neq 0$. The higher-dimentional Marcinkiewicz integral μ_Ω is defined by

$$\mu_\Omega(f)(x)=\Big(\int_0^\infty |F_{\Omega,t}(f)(x)|^2\frac{\mathrm{d}t}{t^3}\Big)^{1/2},$$

where

$$F_{\Omega,t}(f)(x)=\int_{|x-y|\leq t}\frac{\Omega(x-y)}{|x-y|^{n-1}}f(y)\mathrm{d}y.$$

Supported by the Natural Science Foundation of Xuzhou Normal University (09XLB02).

The operator μ_Ω is first defined by Stein[1]. Meanwhile, Stein has proved that if Ω is continuous and satisfies the Lip$\alpha(S^{n-1})(0<\alpha\le 1)$ condition

$$|\Omega(x')-\Omega(y')|\le C|x'-y'|^{\alpha}, \qquad \forall x',y'\in S^{n-1},$$

then μ_Ω is an operator of strong type $(p,p)(1<p\le 2)$ and of weak type $(1,1)$. In [2], it is proved that if $\Omega\in C^1(S^{n-1})$, then μ_Ω is bounded on $L^p(\mathbf{R^n})$ for $1<p<\infty$. The boundedness of μ_Ω have been discussed by many authors(see [3-4] etc).

On the other hand, let $b\in L_{loc}(\mathbf{R^n})$, the commutator μ_Ω^b is defined by

$$\mu_\Omega^b(f)(x)=\Big(\int_0^\infty |F_{\Omega,b,t}(f)(x)|^2\frac{\mathrm{d}t}{t^3}\Big)^{1/2},$$

where

$$F_{\Omega,b,t}(f)(x)=\int_{|x-y|\le t}\frac{\Omega(x-y)}{|x-y|^{n-1}}(b(x)-b(y))f(y)\mathrm{d}y.$$

In this paper $b\in$Lip$_\beta(\mathbf{R^n})\,(0<\beta<1)$, which is the homogeneous Lipschitz space consisting of all functions f such that

$$||f||_{Lip_\beta}=\sup_{x,y\in\mathbf{R^n},x\neq y}\frac{|f(x)-f(y)|}{|x-y|^\beta}<\infty.$$

Obviously, if $b\in$Lip$_\beta(\mathbf{R^n})(0<\beta<1)$, then

$$|b(x)-b(y)|\le C||b||_{\mathrm{Lip}_\beta}|x-y|^\beta \ \ (\forall x,y\in\mathbf{R^n}).$$

Recently,Cheng and Shu[5] considered the commutator μ_Ω^b on Herz-type Hardy spaces, and proved the following theorem.

Theorem A. *Suppose that* $\Omega\in\mathrm{Lip}\nu(S^{n-1})(0<\nu\le 1), b\in\mathrm{Lip}_\beta(\mathbf{R^n})(0<\beta<\min\{1/2,\nu\})$, $0<p<\infty, 1<q_1,q_2<\infty$ *and*

$$1/q_1-1/q_2=\beta/n,\ n(1-1/q_1)\le\alpha<n(1-1/q_1)+\beta,$$

then μ_Ω^b *is bounded from* $H\dot{K}_{q_1}^{\alpha,p}(\mathbf{R^n})$ *to* $\dot{K}_{q_2}^{\alpha,p}(\mathbf{R^n})$.

Lu and Yang[6] introduced the weighted Herz-type Hardy space, and built the atomic decomposition theory. Motivated by [5-6], we consider the weighted boundedness of μ_Ω^b and present our result as follows.

Theorem 1. *Suppose that* $\Omega\in\mathrm{Lip}\nu(S^{n-1})(0<\nu\le 1), b\in\mathrm{Lip}_\beta(\mathbf{R^n})(0<\beta<\min\{1/2,\nu\})$, $0<p_1\le p_2<\infty, 1<q_1,q_2<\infty$ *and*

$$1/q_1-1/q_2=\beta/n,\ n(1-1/q_1)\le\alpha<n(1-1/q_1)+\beta,$$

and $\omega_1\in A_1$, $\omega_2^{q_2}\in A_1$,*then* μ_Ω^b *is bounded from* $H\dot{K}_{q_1}^{\alpha,p_1}(\omega_1,\omega_2^{q_1})$ *to* $\dot{K}_{q_2}^{\alpha,p_2}(\omega_1,\omega_2^{q_2})$.