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SOME NEW TYPE OF DIFFERENCE SEQUENCE SPACES DEFINED BY MODULUS FUNCTION AND STATISTICAL CONVERGENCE

Ayhan Esi

(*Adiyaman University, Turkey*)

Binod Chandra Tripathy

(*Institute of Advanced Study in Science and Technology, India)*

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Abstract. In this article we introduce the difference sequence spaces $W_0[f,\Delta_m]$, $W_1[f,\Delta_m]$, $W_\infty[f,\Delta_m]$ and $S[f,\Delta_m]$, defined by a modulus function f. We obtain a relation between $W_1[f,\Delta_m]\cap\ell_\infty[f,\Delta_m]$ and $S[f,\Delta_m]\cap\ell_\infty[f,\Delta_m]$ and prove some inclusion results.

Key words: *strongly Cesàro summable sequence, modulus function, statistical convergence*

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1 Introduction

Throughout the article w, ℓ_∞, c, c_0 denote the spaces of all, bounded, convergent and null sequences respectively. The zero sequence is denoted by $\theta=(0,0,0,\cdots)$.

The notion of difference sequence was introduced by Kizmaz[4] as follows:

$$Z(\Delta)=\{(x_k)\in w:(\Delta x_k)\in Z\},$$

for $Z=\ell_\infty$, c and c_0, where $\Delta x_k=x_k-x_{k+1}$, for all $k\in\mathbf{N}$.

For further investigation see the work [1],[11-15], [17-21].

The notion of modulus function was introduced by Nakano[6] and further investigated by Ruckle[8], Maddox[5], Tripathy and Chandra[16] and many others.

Definition 1.1. A function $f:[0,\infty)\to[0,\infty)$ is called a modulus if

(i) $f(x)=o$ if and only if $x=0$;

(ii) $f(x+y) \leq f(x)+f(y)$;

(iii) f is increasing;

(iv) f is continuous from the right at 0.

It is immediate from (ii) and (iv)that f is continuous everywhere on $[0,\infty)$.

The notion of statistical convergence was introduced by Fast[2] and Schoenberg[9] independently. Later on it was further investigated by Fridy [3], Rath and Tripathy[7], Tripathy[10],[11], Tripathy and Sarma[21], Tripathy and Sen[22] and many others from sequence space point of view and linked with the summability theory. The notion depends on certain density of subsets of N, the set of natural numbers.

Definition 1.2. A subset E of N is said to have density $\delta(E)$ if

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \chi_E(k) \text{ exists},$$

where χ_E is the *characteristic function* of E.

Definition 1.3. A sequence (x_n) is said to be *statistically convergent* to L if for every $\varepsilon > 0$, $\delta(\{k \in N : |x_k - L| \geq \varepsilon\}) = 0$. We write $stat-\lim x_k = L$.

2 Definitions and Preliminaries

Definition 2.1. A sequence space E is said to be solid (or normal) if $(x_k) \in E$ implies $(\alpha_k x_k) \in E$, for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$, for all $k \in N$.

Definition 2.2. A sequence space E is said to be monotone if it contains the canonical preimages of all its step spaces.

Remark 2.1. It is clear from the above two definitions that “if a sequence space E is solid, then it is monotone".

Definition 2.3. A sequence space E is said to be convergence free if $(y_k) \in E$ whenever $(x_k) \in E$ and $y_k = 0$ whenever $x_k = 0$.

Definition 2.4. A sequence space E is said to be *symmetric* if $(x_{\pi(n)}) \in E$, whenever $(x_n) \in E$, where π is a permutation of N.

Definition 2.5. A sequence space E is said to be *convergence free* if $(y_n) \in E$, whenever $(x_n) \in E$ and $x_n = 0$ implies $y_n = 0$.

Let $m \in N$ be fixed, then the following new type of difference sequence spaces are introduced and studied by Tripathy and Esi[19].

$$Z(\Delta_m) = \{x = (x_k) \in w : (\Delta_m x_k) \in Z\},$$