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MÜNTZ RATIONAL APPROXIMATION FOR SPECIAL FUNCTION CLASSES IN $Ba[0,1]$ SPACES

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Abstract. In this paper,we research the *Müntz* rational approximation of two kinds of special function classes, and give the corresponding estimates of approximation rates of these classes.

Key words: *Müntz rational approximation, bounded variation function class, Sobolev function class, approximation rate*
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1 Introduction

The space Ba introduced by Ding Xiaqi is a new function space[1].

Definition . Let

$$B=\{L_{p_1}[0,1],L_{p_2}[0,1],\cdots,L_{p_m}[0,1],\cdots\}=:\{L_{p_1},L_{p_2},\cdots,L_{p_m},\cdots\}$$

be a sequence of Lebesgue spaces, $p_m>1(m=1,2,\cdots)$, $a=\{a_1,a_2,\cdots,a_m,\cdots\}$ be a nonnegative real number sequence, if for $f(x)\in\bigcap\limits_{m=1}^{\infty}L_{p_m}$, there is a real number $\alpha>0$, such that

$$I(f,\alpha)=\sum_{m=1}^{\infty}a_m\alpha^m\|f\|_{L_{pm}}^m<+\infty,$$

then we say $f(x)\in Ba$, and the norm of Ba is defined by

$$\|f\|_{Ba}=\inf\{\alpha>0:I(f,\frac{1}{\alpha})\leq 1\}. \tag{1.1}$$

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Ba is a Banach space under the norm defined by (1.1)[1].

If we choose $B = \{L_p, L_p, \cdots, L_p, \cdots\}$, $a = \{1, 0, \cdots, 0, \cdots\}$, then we get $I(f, \alpha) = \alpha\|f\|_{L_p}$ and

$$\|f\|_{Ba} = \inf\{\alpha > 0 : I(f, \frac{1}{\alpha}) = \frac{\|f\|_{L_p}}{\alpha} \leq 1\} = \|f\|_{L_p}.$$

In this paper, we always suppose that $p_0 = \inf_m \{p_m\} > 1$, and denote

$$s = \inf_{m \geq 1}\{a_m^{\frac{1}{m}}\}, \quad q = \sup_{m \geq 1}\{a_m^{\frac{1}{m}}\}.$$

For convenience , we denote

$$\|f\|_p = \|f\|_{L_p}, \qquad 1 \leq p < +\infty,$$

$$\|f\|_\infty = \|f\|_C = \max_{0 \leq t \leq 1} |f(t)|, \qquad p = \infty.$$

C always denotes an absolutely positive constant, and $C(s, q, \cdots)$ denotes a positive constant depending on the letters in the brackets. Their values may be different in different place.

Let $L_p[0,1]$ be the space of all p-power integrable functions on [0,1], $1 \leq p < +\infty$. when $p = +\infty$, it can be considered as $C[0,1]$, that is, the space of all continuous functions on [0,1]. Also, we denote by $AC[0,1]$ all the absolutely continuous functions on [0,1].

For any given real sequence $\{\lambda_n\}_{n=1}^\infty$, denote by $\Pi_n(\Lambda)$ the set of *Müntz* polynomials of degree n, that is, all linear combinations of $\{x^{\lambda_1}, x^{\lambda_2}, \cdots, x^{\lambda_n}\}$, and let $R_n(\Lambda)$ be the *Müntz* rational functions of degree n, that is,

$$R_n(\Lambda) = \{\frac{P(x)}{Q(x)} : P(x), Q(x) \in \Pi_n(\Lambda), Q(x) \geq 0, x \in [0,1]\},$$

if Q(0)=0, we assume that $\lim_{x \to 0^+} \frac{P(x)}{Q(x)}$ exists and is finite.

For $f(x) \in Ba[0,1]$, define the best Müntz rational approximation as

$$R_n(\Lambda)_{Ba} = \inf_{r \in R_n(\Lambda)} \|f - r\|_{Ba}.$$

Our main results are following

Theorem 1. *Assume* $\frac{1}{2} \leq \alpha < +\infty$*, given* $M > 0$*, if* $\lambda_{n+1} - \lambda_n \geq Mn^\alpha$ *for all* $n \geq 1$*, then for any* $f \in BV[0,1]$*, there is a positive constant* $C(s, q, M)$*, such that*

$$R_n(\Lambda)_{Ba[0,1]} \leq C(s, q, M) n^{-\frac{1}{p_0}} V(f).$$

We denote by

$$W_{Ba}^1[0,1] = \{f : f \in AC[0,1], f' \in Ba[0,1]\}$$

the Sobolev function class in Ba space.

Theorem 2. *Assume* $\frac{1}{2} \leq \alpha < +\infty$*, given* $M > 0$*, if* $\lambda_{n+1} - \lambda_n \geq Mn^\alpha$ *for all* $n \geq 1$*, then for any* $f \in W_{Ba}^1[0,1]$*, there is a positive constant* $C(s, q, M, p_0)$*, such that*

$$R_n(f, \Lambda)_{Ba[0,1]} \leq C(s, q, M, p_0) n^{-1} \|f'\|_{Ba[0,1]}.$$