Anal. Theory Appl.
Vol. 28, No. 2 (2012), 111–124
DOI : 10.3969/j.issn.1672-4070.2012.02.002

RECURSIVE REPRODUCING KERNELS HILBERT SPACES USING THE THEORY OF POWER KERNELS

M. Mouattamid

(*University of Banjia Luka, Bosnia and Herzegovina)*

Received Mar. 25, 2011

Abstract. The main objective of this work is to decompose orthogonally the reproducing kernels Hilbert space using any conditionally positive definite kernels into smaller ones by introducing the theory of power kernels, and to show how to do this decomposition recursively. It may be used to split large interpolation problems into smaller ones with different kernels which are related to the original kernels. To reach this objective, we will reconstruct the reproducing kernels Hilbert space for the normalized and the extended kernels and give the recursive algorithm of this decomposition.

Key words: *Hilber space, reproducing kernel, interpolant, power function and Conditionally positive kernel*

AMS (2010) subject classification: 46E22, 46B70, 41A05, 47B34, 41D65, 65D05, 65D15

1 Introduction

The abstract theory of Reproducing Kernels Hilbert Space (RKHS) has been developed over a number of years outside of different domains in Physics, Mathematics and/or Chemistry such as the study of conformal mappings[1], integral equations[2], and partial differential equations[3]. The RKHS method has been used for a variety of applications, especially in data interpolation and smoothing[4−7]. The RKHS method provides a rigorous and effective framework for smooth multivariate interpolation of arbitrarily scattered data and for accurate approximation of general multidimensional functions using conditionally/unconditionally positive kernels. Smooth global multi-dimensional reproducing kernels have been successfully used in other contexts for multivariate interpolation, e.g., in computer aided geometric design [8,9] and to solve differential equations by collocation[10]. These reproducing kernels usually are simple and easily to compute in closed forms[10,11]. The reproducing property imparts a rich physically based structure in the associated Hilbert space that possesses many important properties (e.g., the uniqueness and positive definiteness of the reproducing kernel which are important for its practical utility).

The association of a Hilbert space to each conditionally positive definite function go back to the analysis of Madych[13]. The practical advantage of all of this is that all useful conditionally positive definite functions, which were constructed without any relation to an Hilbert space, can be investigated thoroughly within their native space, once the latter is defined and characterized. RKHS, in the conditionally positive definite case, turns out to be a Hilbert space plus a finite-dimensional space[12,17,19].

Section 2 will summarize the recent work in the construction of RKHS (will be called native space) for the conditionally positive kernels Φ and also introduce the power kernels and its native space [15]. Section 3 will present the construction of RKHS for the normalized kernel[18]. Section 4 will introduce an extended kernel Φ_P of the normalized kernel that have the same RKHS. We will show the condition where the interpolation to Φ_P does coincide with the one associated to Φ. Section 5 is the core of this work. The main idea is to decompose large interpolation problems into smaller ones using the theory of power kernels and its RKHS. The orthogonal decomposition of the original native Hilbert space, involving the native space of the power kernel which is proven in our previous work[15]. We will show how to do this orthogonal decomposition of RKHS recursively. It turns out to be used to split large interpolation problems into smaller ones with different kernels which is related to the original kernels Φ.

2 Native Space for the Power Kernels

The interpolation, of scattered data $(x_i, f_i) \in \mathbf{R}$ for pairwise points of discrete set $X = \{x_1, \cdots, x_N\}$ and real valued data $f(x_1), \cdots, f(x_N)$, uses a symmetric multivariate function $\Phi : \mathbf{R}^d \times \mathbf{R}^d \to \mathbf{R}$ for all $x, y \in \mathbf{R}^d$ and the Q-dimensional space $\mathbf{P}_m^d$ of polynomials p_k on $\mathbf{R}^d$ of degree m, to construct the interpolant:

$$s(x) = \sum_{j=1}^{N} \alpha_j \Phi(x, x_j) + \sum_{k=1}^{Q} \beta_k p_k(x) \quad \text{where} \quad x \in \mathbf{R}^d, \tag{2.1}$$

where α_i and β_i are real numbers, via the system

$$\begin{cases} \sum_{j=1}^{N} \alpha_j \Phi(x, x_j) + \sum_{k=1}^{Q} \beta_k p_k(x) = f_i, \\ \sum_{j=1}^{N} \alpha_j p_k(x_j) = 0, \end{cases} \tag{2.2}$$