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COMPLETE HYPERSURFACES WITH CONSTANT SCALAR CURVATURE IN A SPECIAL KIND OF LOCALLY SYMMETRIC MANIFOLD

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Abstract. In this paper, we investigate n-dimensional complete and orientable hypersurfaces M^n $(n \geq 3)$ with constant normalized scalar curvature in a locally symmetric manifold. Two rigidity theorems are obtained for these hypersurfaces.

Key words: *hypersurfaces, scalar curvture, locally symmetric manifold*

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1 Introduction

When the ambient manifolds possess very nice symmetry, for example the unit sphere, there are many rigidity results for hypersurfaces with constant mean curvature or with constant scalar curvature in these ambient manifolds, such as [2, 3, 4, 6, 7, 11, 12] and the references therein. Recently, many researchers studied the minimal hypersurfaces or hypersurfaces with constant mean curvature in more general Riemmanniam manifolds such as the locally symmetric manifolds and the δ-pinched manifolds, and obtained many rigidity results these hypersurfaces, such as [5, 10, 13, 14] and the references therein. It is natural and very important to study n-dimensional complete and orientable hypersurfaces with constant scalar curvature in a locally symmetric manifold. In the paper, we will discuss complete hypersurfaces in this direction.

In order to represent our theorems, we need some notation. Let N^{n+1} be a locally symmetric manifold and M^n be an n-dimensional complete and oriented hypersurface in N^{n+1}. We choose a

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local orthonormal frame $e_1,\cdots,e_n,e_{n+1}$ in N^{n+1} such that $e_1,\cdots,e_n$ are tangent to M^n and e_{n+1} is normal to M^n. We assume that N^{n+1} satisfies the following conditions:

$$K_{n+1in+1i} = c_0, \tag{1}$$

$$\frac{1}{2} < \delta \leq K_N \leq 1, \tag{2}$$

where c_0, δ are constants and K_N denotes the sectional curvature of N^{n+1}. When N^{n+1} satisfies the above conditions (1), (2), it is said simply for N^{n+1} to satisfy the condition $(*)$.

Remark 1.1. If N^{n+1} is a unit sphere $S^{n+1}(1)$, then it satisfies the condition $(*)$, where $c_0 = \delta = 1$.

It is easy to know that the scalar curvature $\bar{R}$ of locally symmetric manifold is constant. On the other hand, if we denote $\bar{R}_{CD}$ as the components of the Ricci curvature tensor of N^{n+1} satisfying the condition $(*)$, then the scalar curvature $\bar{R}$ of N^{n+1} is

$$\bar{R} = 2\sum_k K_{n+1kn+1k} + \sum_{ij} K_{ijij} = 2nc_0 + \sum_{ij} K_{ijij}, \tag{3}$$

hence, $\sum_{ij} K_{ijij}$ is constant. This fact together with the formula (12) suggests us to define a constant P by

$$n(n-1)P = n(n-1)R - \sum_{ij} K_{ijij} = n^2H^2 - S. \tag{4}$$

Using (4), we finally establish our main results:

Theorem 1.2. *Let M^n $(n \geq 3)$ be an n-dimensional complete and orientable hypersurface with constant normalized scalar curvature R in a locally symmetric manifold N^{n+1} satisfying the condition $(*)$. If $P \geq 0$, in the case where $P = 0$, assume further that the mean curvature function H does not change sign, then*

(i) *either* $\sup|\Phi|^2 = 0$ *and M is a totally umbilical hypersurface.*

(ii) *or*

$$\sup|\Phi|^2 \geq D(n,P) = \frac{n(n-1)(P+c)^2}{(n-2)(nP+2c)} > 0. \tag{5}$$

Moreover, if $P > 0$ the equality $\sup|\Phi|^2 = D(n,P)$ *holds and this suppremum is attained at some point of M, then M^n has two distinct constant principal curvatures, one of them being simple, where $c = 2\delta - c_0 > 0$ and P determined by* (4).