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EXISTENCE PROBLEMS OF ADDITIVE SELECTION MAPS FOR ANOTHER TYPE SUBADDITIVE SET-VALUED MAP

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Abstract. In this paper, we consider the following subadditive set-valued map $F : X \longrightarrow P_0(Y)$:

$$F\big(\sum_{i=1}^{r} x_i + \sum_{j=1}^{s} x_{r+j}\big) \subseteq rF\big(\frac{\sum_{i=1}^{r} x_i}{r}\big) + sF\big(\frac{\sum_{j=1}^{s} x_{r+j}}{s}\big), \quad \forall x_i \in X, \quad i = 1, 2, \cdots, r+s,$$

where r and s are two natural numbers. And we discuss the existence and unique problem of additive selection maps for the above set-valued map.

Key words: *additive selection map, subadditive, additive selection, cone*

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1 Introduction and Preliminaries

The stability problem of functional equations was originated from a question of Ulam[1] concerning the stability of group homomorphisms. In 1941, D.H Hyers[2] gave a first affirmative partial answer to the question of Ulam for Banach spaces. The famous stability theorem is as follows:

Theorem 0. *Let E_1 be a normed vector space and E_2 a Banach space. Suppose that the mapping $f : E_1 \to E_2$ satisfies the inequality*

$$\|f(x+y) - f(x) - f(y)\| \le \varepsilon \tag{0}$$

for all $x, y \in E_1$, with $\varepsilon > 0$ a constant. Then the limit

$$g(x) = \lim_{n\to\infty} 2^{-n} f(2^n x)$$

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exists for each $x \in E_1$ *and* g *is the unique additive mapping satisfying*

$$\|f(x) - g(x)\| \le \varepsilon$$

for all $x \in E_1$.

Later, Hyers' Theorem has been generalized by many authors[3−8].

Let X a real vector space. We denote by $P_0(X)$ the family of all nonempty subsets of X.

If Y is a topological vector space, the family of all closed convex subsets of Y denoted by $\mathrm{ccl}(Y)$.

Let A and B are two nonempty subsets of the real vector space X, λ and μ are two real numbers. Define

$$A + B = \{x | x = a + b, a \in A, b \in B\};$$
$$\lambda A = \{x | x = \lambda a, a \in A\}.$$

The next properties are obvious:

Lemma. *If* A *and* B *are two nonempty subsets of the real vector space* X*,* λ *and* μ *are two real numbers, then*

$$\lambda(A + B) = \lambda A + \lambda B; \qquad (\lambda + \mu)A \subseteq \lambda A + \mu A.$$

Furthermore, if A *is a convex subset and* $\lambda\mu \ge 0$*, then we have the following formula*[9]*:*

$$(\lambda + \mu)A = \lambda A + \mu A.$$

A subset $A \subset X$ is said to be a cone if $A + A \subseteq A$, and $\lambda A \subseteq A$ for all $\lambda > 0$.

If the zero in X belongs to A, we say that A is a zero cone.

Let X and Y be two real vector spaces, $f : X \longrightarrow Y$ a single-valued map, and $F : X \longrightarrow P_0(Y)$ a set-valued map. f is called an additive selection of F, if $f(x+y) = f(x) + f(y)$ for all $x, y \in X$, and $f(x) \in F(x)$ for all $x \in X$.

Let $B(0, \varepsilon)$ denote the open ball with center 0 and radius ε in E_2 in Theorem 0, then the inequality (0) may be written as

$$f(x + y) \in B(0, \varepsilon) + f(x) + f(y),$$

and hence

$$f(x + y) + B(0, \varepsilon) \subseteq f(x) + B(0, \varepsilon) + f(y) + B(0, \varepsilon).$$

where $B(0, \varepsilon) + x$ denote the open ball with center x and radius ε in E_2.

Thus, if we define a set-valued mapping F by $F(x) = f(x) + B(0, \varepsilon)$ for each $x \in E_1$, then we get

$$F(x + y) \subseteq F(x) + F(y)$$

and

$$g(x) \in F(x)$$