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MONOTONE POINTS IN ORLICZ-BOCHNER SEQUENCE SPACES

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Abstract. In Orlicz-Bochner sequence spaces endowed with Orlicz norm and Luxemburg norm, points of lower monotonicity, upper monotonicity, lower local uniform monotonicity and upper local uniform monotonicity are characterized.

Key words: *Banach lattice, Orlicz-Bochner space, Luxemburg norm, Orlicz norm, upper (lower) monotone point, upper (lower) locally uniformly monotone point*

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1 Introduction

A Banach lattice X with a lattice norm $\|\cdot\|$ is said to be strictly monotone if for any $x\in X^+$(positive cone in X) and any $y\in X^+\backslash\{0\}$, we have $\|x+y\|>\|x\|$. A point $x\in S(X^+):=S(X)\cap X^+$ is said to be upper monotone (UM for short) if for any $y\in X^+\backslash\{0\}$, $\|x+y\|>1$. A point $x\in S(X^+)$ is said to be lower monotone (LM) if for any $y\in X^+\backslash\{0\}$ and $y\leq x$, $\|x-y\|<1$. An equivalent condition for X being strictly monotone[15] is that any point $x\in S(X^+)$ is lower monotone. But lower monotone points and upper monotone points are different, see [12]. X is called upper locally uniformly monotone[10] if for any $\varepsilon>0$ and $x\in S(X^+)$, there exists $\delta(x,\varepsilon)>0$ such that $y\in X^+$ and $\|y\|\geq\varepsilon$ imply $\|x+y\|\geq 1+\delta(x,\varepsilon)$. If for any $\varepsilon>0$ and $x\in S(X^+)$, there is $\delta(x,\varepsilon)>0$ such that $\|x-y\|\leq 1-\delta(x,\varepsilon)$ whenever $y\in X^+$, $\|y\|\geq\varepsilon$ and $y\leq x$, then X is said to be lower locally uniformly monotone[10]. We can analogously

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define point of lower local uniform monotonicity (LLUM) and point of upper local uniform monotonicity(ULUM).

It is well known that some rotundity properties of Banach spaces have been widely applied in ergodic theory, fixed pointed theory, probability theory and approximation theory etc, and in many cases these rotundity properties can be replaced by respective monotonicity properties when restrict ourselves to Banach space being Banach lattice[1, 2, 3, 10]. Moreover, there are close relationships between monotonicity points and rotundity points[7]. Therefore in recent years monotonicity properties and monotonicity points have been widely investigated in Musielak-Orlicz, Orlicz-Lorentz, Köthe-Bochner, Calderón-Lozanovskiĭ spaces[5,9−14,16]. In this paper we mainly give the criteria for a point in Orlicz-Bochner sequence spaces being UM, LM, ULUM, and LLUM.

Let $\mathbf{R},\mathbf{N}$ stand for the set of all real and natural numbers respectively. A function $M:\mathbf{R}\to\mathbf{R}^+$ is called a $\mathcal{N}-$function if M is convex, even, $M(0)=0$, $M(u)>0$ $(u\neq 0)$ and $\lim\limits_{u\to 0}\dfrac{M(u)}{u}=0$, $\lim\limits_{u\to\infty}\dfrac{M(u)}{u}=\infty$. M is said to satisfy the δ_2-condition for small u $(M\in\delta_2)$ if for some K and $u_0>0$, $M(2u)\leq KM(u)$ as $|u|\leq u_0$.

In the sequel, M and N denote a pair of complemented $\mathcal{N}-$ functions, p the right-hand derivative of M. For a real sequence $u=(u(1),u(2),\cdots)$, we call $\rho_M(u)=\sum_{i=1}^{\infty}M(u(i))$ the modular of u. The linear space $\{u:\rho_M(\lambda u)<\infty \text{ for some } \lambda>0\}$ equipped with the Orlicz norm

$$\|u\|_M=\sup_{\rho_N(v)\leq 1}\sum_{i=1}^{\infty}u(i)v(i)=\inf_{k>0}\frac{1}{k}\{1+\rho_M(ku)\}=\frac{1}{k}\{1+\rho_M(ku)\},\qquad \forall k\in K(u),$$

where $K(u)=[k^*,k^{**}]$, $k^*=\inf\{k>0:\rho_N(p(k|u|))\geq 1\}$, $k^{**}=\sup\{k>0:\rho_N(p(k|u|))\leq 1\}$, or the Luxemburg norm

$$\|u\|_{(M)}=\inf\left\{\lambda>0:\rho_M\left(\frac{u}{\lambda}\right)\leq 1\right\}$$

are Banach spaces. They are called Orlicz sequence spaces, and denoted by l_M and $l_{(M)}$ respectively. Their subspace $h_M=\{u:\rho_M(\lambda u)<\infty \text{ for all } \lambda>0\}$ endowed with the norms, denoted by h_M and $h_{(M)}$ respectively, are also Banach spaces.

If $u=(u(1),u(2),\cdots)$, where $u(i)\in X_i$ and X_i is a Banach space for any $i\in\mathbf{N}$, we denote by $l_M(X_i)$ and $l_{(M)}(X_i)$. We call such spaces Orlicz-Bochner sequence spaces. Moreover, we have known that if X_i is a Banach lattice for each $i\in\mathbf{N}$, then $l_M(X_i)$ and $l_{(M)}(X_i)$ are Banach lattices (with $u\leq v$ if and only if $x(i)\leq y(i)$) as well. In this paper, for $u=(u(1),u(2),\cdots)$, denote $S_u:=\{i\in\mathbf{N}:u(i)\neq 0\}$. For more details about Orlicz and Orlicz-Bochner spaces, we refer to [4, 8, 14].