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ASYMPTOTIC BEHAVIOR OF THE ECKHOFF APPROXIMATION IN BIVARIATE CASE

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Abstract. The paper considers the Krylov-Lanczos and the Eckhoff approximations for recovering a bivariate function using limited number of its Fourier coefficients. These approximations are based on certain corrections associated with jumps in the partial derivatives of the approximated function. Approximation of the exact jumps is accomplished by solution of systems of linear equations along the idea of Eckhoff. Asymptotic behaviors of the approximate jumps and the Eckhoff approximation are studied. Exact constants of the asymptotic errors are computed. Numerical experiments validate theoretical investigations.

Key words: *Krylov-Lanczos approximation, Eckhoff approximation, Bernoulli polynomials, convergence acceleration*

AMS (2010) subject classification: 42A10, 65T40

1 Introduction

It is well known that approximation of a 2-periodic and smooth function on the real line by the truncated Fourier series

$$S_N(f;x) := \sum_{n=-N}^{N} f_n e^{i\pi nx}, \ f_n := \frac{1}{2}\int_{-1}^{1} f(x)e^{-i\pi nx}\mathrm{d}x$$

is highly effective. If the 2-periodic extensions of f and its derivatives up to the order p are continuous on the real line, but $f^{(p)}$ is discontinuous, the uniform convergence rate is $O(N^{-p})$ (see[48]). The approximation is accompanied by the Gibbs phenomenon[48] when the approximated function is discontinuous or non-periodic. The oscillations caused by this phenomenon typically propagate into regions away from singularities and degrade the quality of approximation. Even if the approximated function is analytic but non-periodic the error falls only as fast as

$O(1/N)$ over the entire interval except in zones of width $O(1/N)$ near $x = \pm 1$ where the error is always $O(1)$.

Much efforts were devoted to overcoming the convergence deficiency (see, for example, [3], [11], and [45] with references therein). An efficient approach of convergence acceleration by subtracting a polynomial representing the discontinuities (jumps) in the function and its derivatives was suggested by Krylov in 1906[23] (see also [40] (pp. 88–99), and [46] (pp. 144-147)). Lanczos, in 1956 [24], in 1964[25] and in 1966[26] independently developed the same approach with more formality. He introduced a basic system of polynomials $B(k;x)$ that played a central role in the method and pointed out a close connection between the $B(k;x)$ and Bernoulli polynomials. That was why polynomial subtraction method was called also as Bernoulli method. Jones and Hardy in 1970 [22] and Lyness in 1974[29] considered convergence acceleration of trigonometric interpolation by polynomial subtraction. They showed the relation of the Krylov-Lanczos method with the theory of Lidstone interpolation[27]. Since then, it widely considered in the context of Fourier series[3],[8]−[11],[19],[31],[32], and trigonometric interpolation[32],[38].

The key problem in the Krylov-Lanczos method is approximation of the exact jump values. Ordinarily, such values are unknown and in general only Fourier coefficients or discrete Fourier coefficients of a given function may be specified. If arbitrary pointwise values of the function can be calculated then the finite difference formulas can be set up for approximation of these quantities. Approaches resembling this approach have been attempted under various names and apparently with varying success for the special case where the approximated function is smooth with only singularity at the end points of the interval: Gottlieb and Orszag in 1977 (polynomial subtractions for nonperiodic problems)[20], Lanczos in 1966 (increasing the convergence of Fourier series by adding properly chosen boundary terms)[26] Lyness in 1974 (Lanczos representation)[29], and Roache in 1978 (reduction to periodicity)[41]. Whereas, even if the arbitrary pointwise values of the function can be calculated, approximation of jump values via finite differences is not recommended for this purpose [29]. Even in the case of a uniform grid, finite difference approximations are notoriously unreliable. Moreover, in many applications the Fourier coefficients can be calculated but pointwise values and derivatives are not explicitly available.

As noted in[15], the previous lack of robust methods for the approximation of jump values was the central reason why the polynomial subtraction technique has not been utilized more extensively. The first attempt towards more robust approach was initiated by Gottlieb et al[21]