Analysis in Theory and Applications
Anal. Theory Appl., Vol. **29**, No. 1 (2013), pp. 21-26
DOI: 10.4208/ata.2013.v29.n1.3

New Criterion for Starlike Integral Operators

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Received 30 May 2010

Abstract. In this paper, we introduce new sufficient conditions for certain integral operators to be starlike and p-valently starlike in the open unit disk.

Key Words: Analytic function, univalent function, p-valently starlike, integral operator.

AMS Subject Classifications: 30C45

1 Introduction

Let $\mathcal{U}=\{z\in\mathbf{C}:|z|<1\}$, the unit disk. We denote by $\mathcal{H}(\mathcal{U})$ the class of holomorphic functions defined on $\mathcal{U}$. Let $\mathcal{A}_p$ be the class of all p-valent analytic functions of the form

$$f(z)=z^p+a_{p+1}z^{p+1}+\cdots,\quad p\in\mathbf{N}=\{1,2,\cdots\}.$$

For $p=1$, we obtain $\mathcal{A}_1=\mathcal{A}$, the class of univalent analytic functions in the unit disk. Let $\mathcal{S}^*$ and $\mathcal{K}$ denote the subclasses of starlike and convex functions in $\mathcal{U}$ respectively. Recall that $f\in\mathcal{A}$ is convex if and only if

$$\mathrm{Re}\Big(\frac{zf''(z)}{f'(z)}+1\Big)>0,\quad z\in\mathcal{U},$$

and starlike if and only if

$$\Re\Big(\frac{zf'(z)}{f(z)}\Big)>0,\quad z\in\mathcal{U}.$$

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For $f_i(z)\in\mathcal{A}$ and $\alpha_i>0$, for all $i\in\{1,2,3,\cdots,n\}$, D. Breaz and N. Breaz [2] introduced the following integral operator:

$$F_n(z)=\int_0^z\Big(\frac{f_1(t)}{t}\Big)^{\alpha_1}\cdots\Big(\frac{f_n(t)}{t}\Big)^{\alpha_n}dt. \tag{1.1}$$

Recently Breaz et al. in [3] introduced the following integral operator:

$$F_{\alpha_1,\cdots,\alpha_n}(z)=\int_0^z[f_1'(t)]^{\alpha_1}\cdots[f_n'(t)]^{\alpha_1}dt. \tag{1.2}$$

The most recent, Frasin [1] introduced the following integral operators, for $\alpha_i>0$ and $f_i\in\mathcal{A}_p$,

$$F_p(z)=\int_0^z pt^{p-1}\Big(\frac{f_1(t)}{t^p}\Big)^{\alpha_1}\cdots\Big(\frac{f_n(t)}{t^p}\Big)^{\alpha_n}dt \tag{1.3}$$

and

$$G_p(z)=\int_0^z pt^{p-1}\Big(\frac{f_1'(t)}{pt^{p-1}}\Big)^{\alpha_1}\cdots\Big(\frac{f_1'(t)}{pt^{p-1}}\Big)^{\alpha_n}dt. \tag{1.4}$$

Remark 1.1. (i) For $p=1$, we get $F_1(z)=F_n(z)$, and $G_1(z)=F_{\alpha_1,\cdots,\alpha_n}(z)$.

(ii) For $p=n=1$, $\alpha_1=\alpha\in[0,1]$ in (1.3) we get the integral operator

$$F_\alpha(z)=\int_0^z\Big(\frac{f(t)}{t}\Big)^{\alpha}dt,$$

which is studied in [7].

(iii) For $p=n=1$, $\alpha=1$ in (1.3) we get the integral operator

$$G(z)=\int_0^z\frac{f(t)}{t}$$

introduced by Alexander [4].

(iv) For $p=n=1$, $\alpha_1=\alpha\in\mathbf{C}$, $|\alpha|\le 1/4$ in (1.4) we get the integral operator

$$\int_0^z(f'(t))^{\alpha}dt,$$

which is studied in [5].

2 Main result

In order to prove our main results we shall need the following lemma due to S. S. Miller and P. T. Mocanu [6]: