Analysis in Theory and Applications
Anal. Theory Appl., Vol. **29**, No. 1 (2013), pp. 27-36
DOI: 10.4208/ata.2013.v29.n1.4

Order of Magnitude of Multiple Fourier Coefficients

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Received 26 May 2011

Abstract. The order of magnitude of multiple Fourier coefficients of complex valued functions of generalized bounded variations like $(\Lambda^1,\cdots,\Lambda^N)BV^{(p)}$ and $r-BV$, over $[0,2\pi]^N$, are estimated.

Key Words: Order of magnitude of multiple Fourier coefficients, function of $(\Lambda^1,\cdots,\Lambda^N)BV^{(p)}$, $r-BV$ and $\text{Lip}(p;\alpha_1,\cdots,\alpha_N)$.

AMS Subject Classifications: 42B05, 26B30, 26D15

1 Introduction

Recently, V. Fülöp and F. Móricz [3] studied the order of magnitude of multiple Fourier coefficients of functions in BV ($\overline{\mathbf{T}}^N$), where $\mathbf{T}=[0,2\pi)$, in the sense of Vitali and Hardy. Here, we have generalized these results by estimating the order of magnitude of multiple Fourier coefficients of complex valued functions in $(\Lambda^1,\cdots,\Lambda^N)BV^{(p)}$, $r-BV$ and $\text{Lip}(p;\alpha_1,\cdots,\alpha_N)$ over $\overline{\mathbf{T}}^N$.

Definition 1.1. For a given $f\in L^p(\overline{\mathbf{T}}^2)$, $1\le p<\infty$, the p-integral modulus of continuity of f is defined as

$$\omega^{(p)}(f;\delta_1,\delta_2)=\sup\left\{\left(\frac{1}{4\pi^2}\int\int_{\overline{\mathbf{T}}^2}|\Delta f(x,y;h,k)|^p dxdy\right)^{1/p}:\ 0<h\le\delta_1,\ 0<k\le\delta_2\right\},$$

where

$$\Delta f(x,y;h,k)=f(x+h,y+k)-f(x,y+k)-f(x+h,y)+f(x,y).$$

For every $f\in L^p(\overline{\mathbf{T}}^2)$, $\omega^{(p)}(f;\delta_1,\delta_2)\to 0$ as $\max\{\delta_1,\delta_2\}\to 0$.

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For $p\geq 1$ and $\alpha_1,\alpha_2\in(0,1]$, we say that $f\in \mathrm{Lip}(p;\alpha_1,\alpha_2)$ if

$$\omega^{(p)}(f;\delta_1,\delta_2)=\mathcal{O}(\delta_1^{\alpha_1}\delta_2^{\alpha_2}) \text{ as } \delta_1 \text{and } \delta_2\to 0.$$

For $p=\infty$, we write $\omega(f;\delta_1,\delta_2)$ for $\omega^{(\infty)}(f;\delta_1,\delta_2)$, Definition 1.1 gives the modulus of continuity of f and in that case the class $\mathrm{Lip}(p;\alpha_1,\alpha_2)$ reduces to Lipschitz class $\mathrm{Lip}(\alpha_1,\alpha_2)$.

Definition 1.2. Let **L** be the class of all non-decreasing sequences $\Lambda'=\{\lambda'_n\}$ $(n=1,2,\cdots)$ of positive numbers such that $\sum_n(\lambda'_n)^{-1}$ diverges. For given $\bigwedge=(\Lambda^1,\Lambda^2)$, where $\Lambda^k=\{\lambda^k_n\}\in\mathbf{L}$ for $k=1,2$ and $p\geq 1$. A complex valued measurable function f defined on a rectangle $R:=[a,b]\times[c,d]$ is said to be of p-(Λ^1,Λ^2)-bounded variation (that is, $f\in(\Lambda^1,\Lambda^2)BV^{(p)}(R)$), if

$$V_{\bigwedge_p}(f,R)=\sup_{P=P_1\times P_2}\Big(\sum_{i=1}^{m}\sum_{j=1}^{l}\frac{|\Delta f(x_i,y_j)|^p}{\lambda^1_i\lambda^2_j}\Big)^{1/p}<\infty,$$

where

$$\Delta f(x_i,y_j)=\Delta f(x_i,y_j;\Delta x_i,\Delta y_j),\qquad \Delta x_i=x_{i+1}-x_i,$$
$$\Delta y_j=y_{j+1}-y_j,\qquad P_1: a=x_0<x_1<x_2<\cdots<x_m=b$$

and

$$P_2: c=y_0<y_1<y_2<\cdots<y_l=d.$$

If $f\in(\Lambda^1,\Lambda^2)BV^{(p)}(R)$ is such that the marginal functions $f(a,\cdot)\in\Lambda^2BV^{(p)}([c,d])$ and $f(\cdot,c)\in\Lambda^1BV^{(p)}([a,b])$ (refer [6]) for the definition of $\Lambda BV^{(p)}([a,b])$), then f is said to be of p-$(\Lambda^1,\Lambda^2)^*$-bounded variation over R (that is, $f\in(\Lambda^1,\Lambda^2)^*BV^{(p)}(R)$).

If $f\in(\Lambda^1,\Lambda^2)^*BV^{(p)}(R)$ then f is bounded and each of the marginal function $f(\cdot,t)\in\Lambda^1BV^{(p)}([a,b])$ and $f(s,\cdot)\in\Lambda^2BV^{(p)}([c,d])$, where $t\in[c,d]$ and $s\in[a,b]$ are fixed.

Note that, for $\Lambda^1=\Lambda$ and $\Lambda^2=\{1\}$ (that is, $\lambda^1_n=\lambda_n$ and $\lambda^2_n=1$, $\forall n$) the class $(\Lambda^1,\Lambda^2)BV^{(p)}(R)$ and the class $(\Lambda^1,\Lambda^2)^*BV^{(p)}(R)$ reduce to the class $\Lambda BV^{(p)}(R)$ and the class $\Lambda^*BV^{(p)}(R)$ respectively; for $p=1$, we omit writing p, the class $(\Lambda^1,\Lambda^2)BV^{(p)}(R)$ and the class $(\Lambda^1,\Lambda^2)^*BV^{(p)}(R)$ reduce to the class $(\Lambda^1,\Lambda^2)BV(R)$ (Definition 2, [1]) and the class $(\Lambda^1,\Lambda^2)^*BV(R)$ respectively and for $p=1$ the class $\Lambda BV^{(p)}(R)$ and the class $\Lambda^*BV^{(p)}(R)$ reduce to the class $\Lambda BV(R)$ and the class $\Lambda^*BV(R)$ respectively (Definition 3, [2]). Moreover, for $\Lambda^1=\Lambda^2=\{1\}$ and for $p=1$ the class $(\Lambda^1,\Lambda^2)BV^{(p)}(R)$ and the class $(\Lambda^1,\Lambda^2)^*BV^{(p)}(R)$ reduces to the class $BV_V(R)$ (bounded variation in the sense of Vitali) and the class $BV_H(R)$ (bounded variation in the sense of Hardy) respectively.

Observe that the characteristic function of $E=\{(x,y);x\in[0,1] \text{ and } y\in[0,1-x]\}$ is in $\Lambda BV^{(p)}([0,1]^2)$ if

$$\sum_n\Big(\frac{1}{\lambda_n}\Big)^2<\infty. \tag{1.1}$$