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Boundedness for Commutators of Calderón-Zygmund Operator on Morrey Spaces with Variable Exponent

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Abstract. Our aim in the present paper is to prove the boundedness of commutators on Morrey spaces with variable exponent. In order to obtain the result, we clarify a relation between variable exponent and BMO norms.

Key Words: Commutator, BMO, Morrey space with variable exponent.

AMS Subject Classifications: 42B25

1 Introduction

In 2010, Huang Aiwu, Xu Jingshi [1] have proved the boundedness of multilinear singular integrals and commutators in variable exponent Lebesgue spaces. Recently, Kwok-Pun Ho [2] has proved the boundedness of the fractional integral operators on Morrey spaces with variable exponent on unbounded domains. Motivated by them, we will consider the boundedness of commutators of singular integrals with BMO functions on Morrey spaces with variable exponent.

For any $x\in\mathbb{R}^n$ and $r>0$, write $B(x,r)=\{y\in\mathbb{R}^n:|x-y|<r\}$. χ_B means the characteristic function a measurable set $B\subset\mathbb{R}^n$. In addition, define $\mathbb{B}=\{B(x,r):x\in\mathbb{R}^n,r>0\}$.

In this paper, C always means a positive constant independent of the main parameters and may change from one occurrence to another.

In this section, we recall some definitions.

Definition 1.1 ([1, 3])**.** Let T be a singular integral operator which is initially defined on the Schwartz space $S(\mathbb{R}^n)$. Its values are taken in the space of tempered distributions $S'(\mathbb{R}^n)$ such that for x not in the support of f,

$$Tf(x)=\int_{\mathbb{R}^n}K(x,y)f(y)dy, \tag{1.1}$$

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where f is in $L_c^\infty(\mathbb{R}^n)$, the space of compactly supported bounded functions.

Let $0<\delta, A<\infty$. Here the kernel K is a function in $(\mathbb{R}^n)^2$ away from the diagonal $x=y$ and satisfies the standard estimate

$$|K(x,y)|\leq \frac{A}{|x-y|^n}, \qquad x\neq y, \tag{1.2}$$

and

$$|K(x,y)-K(x',y)|\leq \frac{A|x-x'|^\delta}{(|x-y|+|x'-y|)^{n+\delta}}, \tag{1.3}$$

provided that $|x-x'|\leq \frac{1}{2}\max\{|x-y|,|x'-y|\}$, and

$$|K(x,y)-K(x,y')|\leq \frac{A|y-y'|^\delta}{(|x-y|+|x-y'|)^{n+\delta}}, \tag{1.4}$$

provided that $|y-y'|\leq \frac{1}{2}\max\{|x-y|,|x-y'|\}$.

Such kernel is called a standard kernel and the class of all kernels that satisfy (1.2), (1.3) and (1.4) is denoted by $SK(\delta,A)$.

Let T be as in (1.1) with a kernel in $SK(\delta,A)$. If T is bounded from L^p to L^p with $1<p<\infty$, then we say that T is a Calderón-Zygmund operator.

Let E be a measurable set in $\mathbb{R}^n$ with $|E|>0$.

Definition 1.2 ([4]). Let $p(\cdot):E\to[1,\infty)$ be a measurable function. The Lebesgue space with variable exponent $L^{p(\cdot)}(E)$ is defined by

$$L^{p(\cdot)}(E):=\{f \text{ measurable}: \rho_p(f/\lambda)<\infty \text{ for some constant } \lambda>0\},$$

where $\rho_p(f):=\int_E|f(x)|^{p(x)}dx$.

The space $L^{p(\cdot)}_{loc}(E):=\{f: f\in L^{p(\cdot)}(K) \text{ for all compact subsets } K\subset E\}$.

$L^{p(\cdot)}(E)$ is a Banach space with the norm defined by

$$\|f\|_{L^{p(\cdot)}(E)}:=\inf\{\lambda>0: \rho_p(f/\lambda)\leq 1\}.$$

We denote

$$p_-:=\operatorname{essinf}\{p(x): x\in E\}, \quad p_+:=\operatorname{esssup}\{p(x): x\in E\}.$$

The set $\mathcal{P}(E)$ consists of all $p(\cdot)$ satisfying $p_->1$ and $p_+<\infty$. $p'(\cdot)$ means the conjugate exponent of $p(\cdot)$, namely $1/p(x)+1/p'(x)=1$ holds.

Given a function $f\in L^1_{loc}(E)$, the Hardy-Littlewood maximal operator M is defined by

$$Mf(x):=\sup_{r>0} r^{-n}\int_{B(x,r)\cap E}|f(y)|dy.$$

$\mathcal{B}(E)$ is the set of $p(\cdot)\in\mathcal{P}(E)$ satisfying the condition that M is bounded on $L^{p(\cdot)}(E)$.