Analysis in Theory and Applications
Anal. Theory Appl., Vol. **29**, No. 3 (2013), pp. 289-307
DOI: 10.4208/ata.2013.v29.n3.9

Boundedness of Multilinear Commutators with Rough Kernels on Morrey-Herz Spaces

Qiaoni Li and Shuangping Tao*

College of Mathematics and Information Science, Northwest Normal University, Lanzhou 730070, Gansu, China

Received 15 June 2012; Accepted (in revised version) 23 April 2013

Available online 30 September 2013

Abstract. This paper is concerning the commutators generated by the multilinear singular integral with rough kernels and BMO functions. The boundedness of the multilinear commutators $T_{\vec{b}}(\vec{f})$ is established on the Morrey-Herz space by using the John-Nirenberg inequality.

Key Words: Multilinear operator, rough kernel, commutator, Morrey-Herz space.

AMS Subject Classifications: 42B25, 42B35

1 Introduction

Let $K(x,y_1,\cdots,y_m)$ be a locally integrable function defined away from the diagonal $x=y_1=\cdots=y_m$ in $(\mathbf{R}^n)^{m+1}$. We assume K satisfies the follow size condition

$$|K(x,y_1,\cdots,y_m)|\leq \frac{A}{(|x-y_1|+|x-y_2|+\cdots+|x-y_m|)^{mn}} \tag{1.1}$$

for some $A>0$ and all $(x,y_1,\cdots,y_m)$ with $x\neq y_j$ for some j. Let

$$T:S(\mathbf{R}^n)\times S(\mathbf{R}^n)\times\cdots\times S(\mathbf{R}^n)\to S^{'}(\mathbf{R}^n)$$

be the m-linear operator with the kernel K defined by

$$\begin{aligned}&\langle T(f_1,f_2,\cdots f_m),g\rangle\\ =&\int_{\mathbf{R}^n}\int_{(\mathbf{R}^n)^m}K(x,y_1,\cdots,y_m)f_1(y_1)f_2(y_2)\cdots f_m(y_m)g(x)dy_1dy_2\cdots dy_m dx,\end{aligned} \tag{1.2}$$

*Corresponding author. *Email addresses:* taosp@nwnu.edu.cn (S. P. Tao), nini.881003@163.com (Q. N. Li)

where $f_1,f_2,\cdots,f_m,g\in S(\mathbf{R}^n)$ and $\bigcap_{j=1}^m \text{supp}(f_j)\cap\text{supp}(g)=\emptyset$. The j-th transpose $T^{*,j}$ of T is defined by

$$\langle T^{*,j}(f_1,\cdots f_{j-1},f_j,f_{j+1}\cdots f_m),g\rangle=\langle T(f_1\cdots,f_{j-1},g,f_{j+1},\cdots,f_m),f_j\rangle,$$

where $f_1,f_2,\cdots,f_m,g\in S(\mathbf{R}^n)$. It is easy to check that the kernel $K^{*,j}$ of $T^{*,j}$ is related to that K of T via the identity

$$K^{*,j}(x,y_1,\cdots,y_{j-1},y_j,y_{j+1},\cdots,y_m)=K(y_j,y_1,\cdots,y_{j-1},x,y_{j+1},\cdots,y_m).$$

The operator $\{A_t\}_{t>0}$ are assumed to be associated with kernels $a_t(x,y)$ in the sense that for any $f\in L^p(\mathbf{R}^n)$ with $1<p<\infty$

$$A_t f(x)=\int_{\mathbf{R}^n} a_t(x,y)f(y)dy$$

and the condition

$$|a_t(x,y)|\le h_t(x,y)=t^{-n/s}h\Big(\frac{|x-y|^s}{t}\Big), \tag{1.3}$$

holds, in which s is a positive fixed and his a positive bounded decreasing function satisfying

$$\lim_{r\to\infty} r^{n+\eta}h(r^s)=0, \tag{1.4}$$

for some $\eta>0$.

Assumption 1.1. Assume that for each $i=1,\cdots,m$, there exist operators $\{A_t^{(i)}\}_{t>0}$ with kernels $a_t^{(i)}(x,y)$ that satisfy condition (1.3) and (1.4) with constants s and η, and there exist kernels $K_t^{(i)}(x,y_1\cdots,y_m)$ such that

$$\begin{aligned}&\langle T(f_1,\cdots,A_t^i f_i,\cdots,f_m),g\rangle\\ =&\int_{\mathbf{R}^n}\int_{(\mathbf{R}^n)^m} K_t^{(i)}(x,y_1,\cdots,y_m)f_1(y_1)f_2(y_2)\cdots f_m(y_m)g(x)dy_1dy_2\cdots dy_m dx,\end{aligned}$$

for all $f_i,g\in S(\mathbf{R}^n)$ $(i=1,2,\cdots,m)$ with $\cap_{j=1}^m\text{supp}(f_j)\cap\text{supp}(g)=\emptyset$, and exist a function $\phi\in C(\mathbf{R})$, with supp$\phi\subset[-1,1]$ and a constant $\varepsilon>0$ such that for every $j=0,1,\cdots,m$ and every $i=1,2\cdots,m$, we have

$$\begin{aligned}&|K(x,y_1,\cdots,y_{j-1},y_j,y_{j+1},\cdots,y_m)-K_t^{(i)}(x,y_1,\cdots,y_{j-1},y_j,y_{j+1},\cdots,y_m)|\\ \le&\frac{A}{(|x-y_1|+\cdots+|x-y_m|)^{mn}}\sum_{k=1,k\ne i}^m\phi\Big(\frac{|y_i-y_k|}{t^{1/s}}\Big)+\frac{At^{\varepsilon/s}}{(|x-y_1|+\cdots+|x-y_m|)^{mn+\varepsilon}}\end{aligned}$$

for some $A>0$, where $t^{1/s}\le|x-y_i|/2$.