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On Potentially Graphical Sequences of $G-E(H)$

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Abstract. A loopless graph on n vertices in which vertices are connected at least by a and at most by b edges is called a (a,b,n)-*graph*. A (b,b,n)-*graph* is called (b,n)-*graph* and is denoted by K_n^b (it is a complete graph), its complement by $\overline{K}_n^b$. A non increasing sequence $\pi=(d_1,\cdots,d_n)$ of nonnegative integers is said to be (a,b,n) graphic if it is realizable by an (a,b,n)-graph. We say a simple graphic sequence $\pi=(d_1,\cdots,d_n)$ is potentially $K_4-K_2\cup K_2$-graphic if it has a a realization containing an $K_4-K_2\cup K_2$ as a subgraph where K_4 is a complete graph on four vertices and $K_2\cup K_2$ is a set of independent edges. In this paper, we find the smallest degree sum such that every n-term graphical sequence contains $K_4-K_2\cup K_2$ as subgraph.

Key Words: Graph, (a,b,n)-*graph*, potentially graphical sequences.

AMS Subject Classifications: 05C07

1 Introduction

Let $G(V,E)$ be a simple graph (a graph without multiple edges and loops) with n vertices and m edges having vertex set $V(G)=\{v_1,v_2,\cdots,v_n\}$. The set of all non-increasing nonnegative integer sequences $\pi=(d_1,d_2,\cdots,d_n)$ is denoted by NS_n. A sequence $\pi\in NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. There are several famous results, Havel and Hakimi [7,8] and Erdös and Gallai [2] which give necessary and sufficient conditions for a sequence $\pi=(d_1,d_2,\cdots,d_n)$ to be the degree sequence of a simple graph G. Another characterization of graphical sequences can be seen in Pirzada and Yin Jian Hu [15]. A graphical sequence π is potentially H-graphical if there is a realization of π containing H as a subgraph, while π is forcibly H graphical if every realization of π contains H as a subgraph. A sequence $\pi=(d_1,d_2,\cdots,d_n)$

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is said to be potentially K_{r+1}- graphic if there is a realization G of π containing K_{r+1} as a subgraph. It is shown in [4] that if π is a graphic sequence with a realization G containing H as a subgraph, then there is a realization G of π containing H with the vertices of H having $|V(H)|$ largest degree of π. If π has a realization in which the $r+1$ vertices of largest degree induce a clique, then π is said to be potentially A_{r+1}-graphic. We know that a graphic sequence π is potentially K_{k+1}-graphic if and only if π is potentially A_{k+1}-graphic [17]. The disjoint union of the graphs G_1 and G_2 is defined by $G_1 \bigcup G_2$. Let K_k and C_k respectively denote a complete graph on k vertices and a cycle on k vertices.

In order to prove our main results, the following notations, definitions and results are needed. Let $G=(V(G),E(G))$ be a simple graph with vertex set $V(G)=\{v_1,v_2,\cdots,v_n\}$. The degree of v_i is denoted by d_i for $1\leq i\leq n$. Then $\pi=(d_1,d_2,\cdots,d_n)$ is the degree sequence of G, where $d_1,d_2,\cdots,d_n$ may be not in increasing order. In order to prove our main results, we also need the following notations and results. Let $\pi=(d_1,d_2,\cdots,d_n)\in NS_n$, $1\leq k\leq n$. Let

$$\begin{aligned} \pi'' &= (d_1-1,\cdots,d_{k-1}-1,\cdots,d_{d_k+1}-1,d_{d_k+2},\cdots,d_n), && \text{if } d_k\geq k,\\ &= (d_1-1,\cdots,d_k-1,\cdots,d_{d_k+1},\cdots,d_{k-1},d_{k+1},d_n), && \text{if } d_k<k. \end{aligned}$$

Denote $\pi'_k=(d_1^{i'}, d_2^{i'},\cdots,d_{n-1}^{i'})$, $1\leq i'\leq n$, where $d_1^{i'},d_2^{i'},\cdots,d_{n-1}^{i'}$ is a rearrangement of the $n-1$ terms of π''. Then π'' is called the residual sequence obtained by laying off d_k from π.

Definition 1.1. A wheel graph W_n is a graph with n vertices $(n\geq 4)$ formed by connecting a single vertex to all vertices of an $(n-1)$ cycle. A wheel graph on 4 and 5 vertices are shown in Fig. 1 below.

In 1960 Erdős and Gallai gave the following necessary and sufficient condition.

Theorem 1.1 (see Erdős, Gallai [2]). *Let $n\geq 1$. An even sequence $\pi=(d_1,\cdots,d_n)$ is graphical if and only if*

$$\sum_{i=1}^{k} d_i \leq k(k-1)+\sum_{i=k+1}^{n}\min(d_i,k)$$

is satisfied for each integer k, $1\leq k\leq n$.

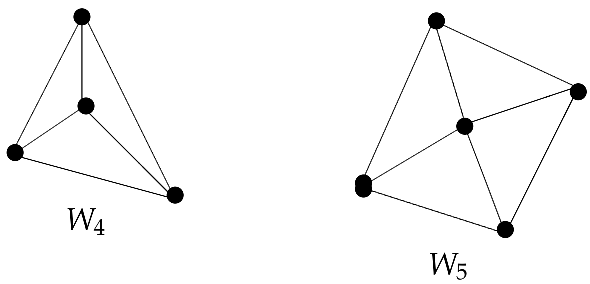

Figure 1: