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Coefficient Estimates for Certain Subclasses of Bi-Univalent Ma-Minda Mocanu-Convex Functions

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Abstract. In this paper, we introduce and investigate a new subclass of the function class Σ of bi-univalent functions of the Mocanu-convex type defined in the open unit disk, satisfy Ma and Minda subordination conditions. Furthermore, we find estimates on the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in the new subclass introduced here. Further Application of Hohlov operator to this class is obtained. Several (known or new) consequences of the results are also pointed out.

Key Words: Analytic functions, univalent functions, bi-univalent functions, bi-starlike functions, bi-convex functions, bi-Mocanu-convex functions, subordination, Hohlov operator.

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1 Introduction

Let $\mathcal{A}$ denote the class of analytic functions of the form

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n \tag{1.1}$$

normalized by the conditions $f(0)=0=f'(0)-1$ defined in the open unit disk $\Delta=\{z\in\mathbb{C}: |z|<1\}$. A function $f\in\mathcal{A}$ is said to be bi-univalent in Δ if both f and f^{-1} are univalent in Δ. Let Σ denote the class of bi-univalent functions defined in the unit disk Δ. Since $f\in\Sigma$

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has the Maclaurin series given by (1.1), a computation shows that its inverse $g=f^{-1}$ has the expansion

$$g(w)=f^{-1}(w)=w-a_2w^2+(2a_2^2-a_3)w^3+\cdots. \tag{1.2}$$

An analytic function f is subordinate to an analytic function g, written as $f(z)\prec g(z)$, provided there is an analytic function w defined on Δ with $w(0)=0$ and $|w(z)|<1$ satisfying $f(z)=g(w(z))$. Ma and Minda [5] unified various subclasses of starlike and convex functions for which either of the quantity $zf'(z)/f(z)$ or $1+zf''(z)/f'(z)$ is subordinate to a more general superordinate function. For this purpose, they considered an analytic function φ with positive real part in the unit disk Δ, $\varphi(0)=1$, $\varphi'(0)>0$, and φ maps Δ onto a region starlike with respect to 1 and symmetric with respect to the real axis. The class of Ma-Minda starlike functions consists of functions $f\in\mathcal{A}$ satisfying the subordination $zf'(z)/f(z)\prec\varphi(z)$. Similarly, the class of Ma-Minda convex functions of functions $f\in\mathcal{A}$ satisfying the subordination $1+zf''(z)/f'(z)\prec\varphi(z)$. A function f is bi-starlike of Ma-Minda type or bi-convex of Ma-Minda type if both f and f^{-1} are respectively Ma-Minda starlike or convex. These classes are denoted respectively by $\mathcal{S}^*_\Sigma(\varphi)$ and $\mathcal{K}_\Sigma(\varphi)$. Also denote by $\mathcal{M}_\Sigma(\lambda,\phi)$ the class of Ma-Minda Mocanu-convex functions consists of functions $f\in\mathcal{A}$ satisfying the subordination

$$(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\Big(1+\frac{zf''(z)}{f'(z)}\Big)\prec\phi(z),\quad \lambda\geq 0.$$

In the sequel, it is assumed that φ is an analytic function with positive real part in the unit disk Δ, satisfying $\varphi(0)=1$, $\varphi'(0)>0$, and $\varphi(\Delta)$ is symmetric with respect to the real axis. Such a function has a series expansion of the form

$$\varphi(z)=1+B_1z+B_2z^2+B_3z^3+\cdots,\quad B_1>0. \tag{1.3}$$

Recently there has been triggering interest to study bi-univalent functions (see [7,9,10]. Motivated by the works of Ali et al. [1] and Goyal and Goswami [3], in this paper we introduce a new subclass $\mathcal{SP}^\gamma_\Sigma(\lambda,h)$ of bi-univalent functions to estimate the coefficients $|a_2|$ and $|a_3|$ for the functions in the class $\mathcal{SP}^\gamma_\Sigma(\lambda,h)$.

Definition 1.1. Let $h:\Delta\to\mathbb{C}$ be a convex univalent function such that $h(0)=1$ and $\Re(h(z))>0$, $z\in\Delta$. A function $f(z)$ is said to be in the class $\mathcal{SP}^\gamma_\Sigma(\lambda,h)$ if the following conditions are satisfied:

$$e^{i\gamma}\Big[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\Big(1+\frac{zf''(z)}{f'(z)}\Big)\Big]\prec h(z)\cos\gamma+i\sin\gamma,\quad f\in\Sigma,\quad z\in\Delta, \tag{1.4}$$

and

$$e^{i\gamma}\Big[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\Big(1+\frac{wg''(w)}{g'(w)}\Big)\Big]\prec h(w)\cos\gamma+i\sin\gamma,\quad w\in\Delta, \tag{1.5}$$

where $\gamma\in(-\pi/2,\pi/2)$, $\lambda\geq 0$ and $g=f^{-1}$.