

Weighted Composition Operators from the Bloch Space to Weighted Banach Spaces on Bounded Homogeneous Domains

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Abstract. We study the bounded and the compact weighted composition operators from the Bloch space into the weighted Banach spaces of holomorphic functions on bounded homogeneous domains, with particular attention to the unit polydisk. For bounded homogeneous domains, we characterize the bounded weighted composition operators and determine the operator norm. In addition, we provide sufficient conditions for compactness. For the unit polydisk, we completely characterize the compact weighted composition operators, as well as provide “computable” estimates on the operator norm.

Key Words: Weighted composition operators, Bloch space, weighted Banach space, homogeneous domain.

AMS Subject Classifications: 47B38, 32A18

1 Introduction

Let X and Y be Banach spaces of holomorphic functions on a domain $\Omega \subset \mathbb{C}^n$. For holomorphic functions $\psi: \Omega \rightarrow \mathbb{C}$ and $\varphi: \Omega \rightarrow \Omega$, the weighted composition operator from X to Y with symbols ψ and φ is defined as

$$W_{\psi, \varphi} f = \psi(f \circ \varphi),$$

for $f \in X$. The weighted composition operator is the generalization of the multiplication operator $M_{\psi} f = \psi f$ and the composition operator $C_{\varphi} f = f \circ \varphi$, called the component operators.

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The study of weighted composition operators on the Bloch space of the unit disk \mathbb{D} began with the work of Ohno and Zhao [16], where the boundedness and compactness were characterized. In higher dimensions, these operators on the Bloch space have been studied on the unit polydisk by Chen, Stević, and Zhou [7], and on bounded homogeneous domains by the author and Colonna [3].

The bounded and compact weighted composition operators from the Bloch space to H^∞ were characterized by Ohno [15] and Hosokawa, Izuchi, and Ohno [11] in the one-dimensional case, and by Li and Stević [13] in the case of the unit ball. In [3], the author and Colonna characterized the boundedness, determined the operator norm, and gave a sufficient condition for compactness in the case of a general bounded homogeneous domain in \mathbb{C}^n .

Composition and multiplication operators on $H_\mu^\infty(\mathbb{D})$ were first studied by Bonet, Domański, Lindström and Taskinen [6], and then later by Bonet, Domański and Lindström [4, 5]. The weighted composition operators on these spaces were studied by Contreras and Hernández-Díaz [8], and by Galindo and Lindström [9].

In [18], Stević determined the norm of the bounded weighted composition operators from the Bloch space to the weighted Banach space H_μ^∞ of the unit ball by means of the more general α -Bloch spaces. Likewise, Yang characterized the bounded and compact weighted composition operators from the Bloch space to H_μ^∞ on the unit ball via the α -Bloch space [22]. Zhu characterized the weighted composition operators from the Bloch space to H_μ^∞ on the unit ball by means of studying the $F(p, q, s)$ spaces, for which the Bloch space is a special case [24]. These operators have not previously been studied for spaces of functions defined on the unit polydisk.

A difficulty in the study of such operators on domains in several variables is the fact that function theory on the unit ball \mathbb{B}_n and the unit polydisk \mathbb{D}^n are vastly different. In recent years, work has been done to generalize these two spaces in an effort to consolidate work. The bounded homogeneous domains are a natural generalization of both \mathbb{B}_n and \mathbb{D}^n , and spaces of holomorphic functions on such domains have been studied. In this paper, we generalize the study of weighted composition operators from the Bloch space to the weighted Banach spaces from \mathbb{B}_n and \mathbb{D}^n to bounded homogeneous domains. This unifies the study of such operators, and allows for study these operators on spaces that are not just \mathbb{B}_n and \mathbb{D}^n , examples of which can be found in [21].

Currently the study of these operators in higher dimensions has taken place in the setting of the unit ball \mathbb{B}_n . This paper introduces to the literature, the study of these operators in the unit polydisk setting. As is normally the case, the techniques used are different than those used for the unit ball. In addition, the author and Colonna studied weighted composition operators from the Bloch space into H^∞ on bounded homogeneous domains in [3]. This paper extends this work to the weighted Banach spaces H_μ^∞ .

In Section 2, we define the Bloch space and ge weighted Banach spaces H_μ^∞ on a bounded homogeneous domain as well as collect useful facts about Bloch functions on such domains. In Section 3, we characterize the bounded weighted composition operator from $\mathcal{B}(D)$ into $H_\mu^\infty(D)$. In addition, we determine the operator norm of such operators.