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Coefficient Estimates for Bi-Univalent Bazilevic Functions

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Abstract. In this paper we consider the class of Bazilevic functions for bi-univalent functions. For this we will estimate the coefficients a_2 and a_3 using Caratheodory functions and the method of differential subordination.

Key Words: Analytic, univalent, unit disk, bi-univalent, Bazilevic functions.

AMS Subject Classifications: 30C45, 30C50

1 Introduction and preliminaries

Let $\mathcal{U}=\{z:|z|\leq 1\}$ be the open unit disk. We consider the class of all functions of the form

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n \tag{1.1}$$

denoted by $\mathcal{A}$, which are analytic in $\mathcal{U}$. We denote by $\mathcal{S}$ the class of all analytical functions that are univalent in $\mathcal{U}$.

Any function $f\in\mathcal{S}$ has an inverse, f^{-1}, that satisfies:

$$f^{-1}(f(z))=z, \quad z\in\mathcal{U},$$

and

$$f(f^{-1}(w))=w, \quad |w|<r_0, \quad r_0(f)\geq\frac{1}{4},$$

(from Koebe One-Quater Theorem, see for example [4], pp. 31). The inverse function of f is given by

$$f^{-1}(w)=w-a_2w^2+(2a_2^2-a_3)w^3-(5a_2^3-5a_2a_3+a_4)w^4+\cdots. \tag{1.2}$$

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A bi-univalent function in $\mathcal{U}$ is a function $f\in\mathcal{A}$, for which f and f^{-1} are both univalent in $\mathcal{U}$. We denote by Σ the class of all bi-univalent functions.

The class of bi-univalent functions was introduced by Lewin in 1967 in [6]. Brannan and Taha in [3] introduced and studied some subclasses of the bi-univalent functions class and estimate the coefficient a_2 and a_3. Also recently Q. Xu et al. in [7] studied a subclass of bi-univalent functions.

We will consider two functions with positive real part, $h(z)$ and $p(z)$ such that $\min\{\mathrm{Re}(h(z)),\mathrm{Re}(p(z))\}>0$ and $h(0)=p(0)=1$. This class of function with positive real part is denoted by $\mathcal{P}$.

We say that an analytical function f is subordinate to an analytical function g if there exists an analytic function h defined on $\mathcal{U}$ with $h(0)=0$ and $|h(z)|<1$ that satisfies $f(z)=g(h(z))$ and we write by $f(z)\prec g(z)$.

We consider ψ an analytic function with positive real part in $\mathcal{U}$, $\psi(0)=1$ and $\psi'(0)>0$. The function ψ has the form

$$\psi(z)=1+Q_1z+Q_2z^2+\cdots,\quad Q_1>0. \tag{1.3}$$

A function $f\in\Sigma$ is in the class $B^{\nu}_{\Sigma}(\psi)$ if we have the following subordination:

$$\frac{zf'(z)}{f(z)}\Big(\frac{f(z)}{z}\Big)^{\nu}\prec\psi(z)$$

and

$$\frac{wg'(w)}{g(w)}\Big(\frac{g(w)}{w}\Big)^{\nu}\prec\psi(w),$$

where $\nu>0$ and $g(w)=f^{-1}(w)$. The functions from the class $B^{\nu}_{\Sigma}(\psi)$ are called bi-univalent Bazilevic functions.

Bazilevic studied the class of Bazilevic type functions in [2] and studied certain properties for this class. To prove our main results we will give the following lemma:

Lemma 1.1 (see [5]). *The coefficient c_n of a function $p\in\mathcal{P}$ satisfy the sharp inequality*

$$|c_n|\le 2,\quad n\ge 1.$$

In this paper we investigate the estimation for the coefficients a_2 and a_3 of the analytic function f motivated by the work of R. Ali et al. in [1].

2 Main results

Theorem 2.1. *If $f\in B^{\nu}_{\Sigma}(\psi)$, $\psi(z)=1+Q_1z+Q_2z^2+\cdots$, $Q_1>0$, $\nu>0$, then*

$$|a_2|\le\frac{Q_1\sqrt{2Q_1}}{\sqrt{|Q_1^2(\nu+2)(\nu+1)+2Q_1(\nu+1)^2-2Q_2(\nu+1)^2|}} \tag{2.1}$$