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Approximation by the Modified q-Baskakov-Szász Operators

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Abstract. In this paper we propose the q analogues of modified Baskakov-Szász operators. We estimate the moments and established direct results in term of modulus of continuity. An estimate for the rate of convergence and weighted approximation properties of the q operators are also obtained.

Key Words: q-analogues, Baskakov-Szász operator, modulus of continuity, weighted approximation.

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1 Introduction

The application of q calculus in approximation theory is one of the main area of research in the last decade. The pioneer work has been made by A. Lupas [11] who introduced a q-anabgue of Bernstein operators $B_{n,q}(f;x)$ and investigated its approximating and shape-preserving property in 1987. See also [14]. Recently, some new q-type extensions of well-known positive linear operators were introduced by several authors. For example, q-Szász-Mirakian operators [1, 13], q-Meyer-König and Zeller operators [3, 16], q-Durrmeyer operators [7] and q-Baskakov operators [2]. In the present article we propose the q analogue of the modified Baskakov-Szász type operators and study the convergence behavior. These operators have better approximation results than the q-Baskakov-Szász type operators studied in [8].

First of all, we recall some concept of q-calculus. All of the results can be found in [6,9]. In what follows, q is a real number satisfying $0<q<1$. For $n\in\mathbb{N}$, the q integer and q factorial are defined as

$$[n]_q=\frac{1-q^n}{1-q},\qquad [n]_q!=\begin{cases}[n]_q[n-1]_q\cdots[1]_q, & n\ge 1,\\ 1, & n=0.\end{cases}$$

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The q-binomial coefficients are given by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[k]_q![n-k]_q!}, \quad 0 \le k \le n.$$

For $t>0$, the q-Gamma integral (see [10]) is defined by

$$\Gamma_q(t) = \int_0^{\frac{1}{1-q}} x^{t-1} E_q(-qx) d_q x, \tag{1.1}$$

where $E_q(x) = \sum_{n=0}^{\infty} q^{n(n-1)/2} \frac{x^n}{[n]_q!}$. Also $\Gamma_q(t+1) = [t]_q \Gamma_q(t)$, $\Gamma_q(1) = 1$.

For $f \in C[0,\infty)$, $q>0$ and each positive integer n, the q-Baskakov operators [2] are defined as

$$\begin{aligned} V_{n,q}(f;x) &= \sum_{k=0}^{\infty} \begin{bmatrix} n+k-1 \\ k \end{bmatrix}_q q^{\frac{k^2-k}{2}} \frac{x^k}{(1+x)_q^{n+k}} f\Big(\frac{[k]_q}{q^{k-1}[n]_q}\Big) \\ &= \sum_{k=0}^{\infty} p_{n,k}^q(x) f\Big(\frac{[k]_q}{q^{k-1}[n]_q}\Big), \end{aligned} \tag{1.2}$$

where

$$(1+x)_q^n = \begin{cases} (1+x)(1+qx)\cdots(1+q^{n-1}), & n=1,2,\cdots, \\ 1, & n=0. \end{cases}$$

Lemma 1.1. *The first three moments of the q-Baskakov operators (see [2]) are given by*

$$V_{n,q}(1;x) = 1, \quad V_{n,q}(t;x) = x, \quad V_{n,q}(t^2;x) = x^2 + \frac{x}{[n]_q}\Big(1 + \frac{1}{q}x\Big).$$

2 Construction of operators

Very recently, Gupta [8] introduced the q-Baskakov-Szász type operators as

$$B_{n,q}(f;x) = [n]_q \sum_{k=0}^{\infty} p_{n,k}^q(x) \int_0^{\frac{q}{1-q^n}} q^{-k-1} s_{n,k}^q(t) f(tq^{-k}) d_q t,$$

where $x \in [0,\infty)$ and

$$p_{n,k}^q(x) = \begin{bmatrix} n+k-1 \\ k \end{bmatrix}_q q^{\frac{k^2-k}{2}} \frac{x^k}{(1+x)_q^{n+k}}, \quad s_{n,k}^q(t) = \frac{([n]_q t)^k}{[k]_q!} E_q(-[n]_q q^k t).$$

Remark 2.1 (see [8])**.** For $B_{n,q}(t^m;x)$, $m=0,1,2$, one has

$$\begin{aligned} &B_{n,q}(1;x) = 1, \quad B_{n,q}(t;x) = x + \frac{q}{[n]_q}, \\ &B_{n,q}(t^2;x) = x^2\Big(1 + \frac{1}{q[n]_q}\Big) + \frac{x}{[n]_q}(1+q(q+2)) + \frac{q^2(1+q)}{[n]_q^2}. \end{aligned}$$