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The Fractional Maximal Operator and Marcinkiewicz Integrals Associated with Schrödinger Operators on Morrey Spaces with Variable Exponent

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Abstract. In this paper, we prove the boundedness of the fractional maximal operator, Hardy-Littlewood maximal operator and marcinkiewicz integrals associated with Schrödinger operator on Morrey spaces with variable exponent.

Key Words: Fractional maximal operator, Marcinkiewicz integrals, Schrödinger, variable exponent, Morrey space.

AMS Subject Classifications: 42B20, 42B35

1 Introduction

In this paper, we consider the Schrödinger differential operator

$$\mathcal{L}=-\Delta+V(x) \quad \text{on } \mathbb{R}^n, \quad n\geq 3,$$

where $V(x)$ is a nonnegative potential belonging to the reverse Hölder class B_q for $q\geq n/2$.

A nonnegative locally L^q integrable function $V(x)$ on $\mathbb{R}^n$ is said to belong to B_q $(q>1)$ if there exists a constant $C>0$ such that the reverse Hölder inequality

$$\Big(\frac{1}{|B|}\int_B V^q dx\Big)^{\frac{1}{q}}\leq C\Big(\frac{1}{|B|}\int_B V dx\Big)$$

holds for every ball in $\mathbb{R}^n$, see [1].

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The Marcinkiewicz integral operator μ is defined by

$$\mu f=\Big(\int_0^\infty\Big|\int_{|x-y|\le t}\frac{\Omega(x-y)}{|x-y|^{n-1}}f(y)dy\Big|^2\frac{dt}{t^3}\Big)^{\frac{1}{2}}.$$

Stein [2] first introduced the operator μ and proved that μ is of type (p,p) $(1<p\le 2)$ and of weak type (1,1) in the case of $\Omega\in Lip_\gamma(S^{n-1})$ $(0<\gamma\le 1)$. Benedek, Calderón and Panzone [3] extended Stein′s results, proved that if $\Omega\in C^1(S^{n-1})$, then μ is of type (p,p) $(1<p<\infty)$.

Similar to the classical marcinkiewicz function μ, one defines the Marcinkiewicz functions μ_j^L associated with the Schrödinger operator L by

$$\mu_j^L f(x)=\Big(\int_0^\infty\Big|\int_{|x-y|\le t}K_j^L(x,y)f(y)dy\Big|^2\frac{dt}{t^3}\Big)^{\frac{1}{2}},$$

where $K_j^L(x,y)=\tilde{K}_j^L(x,y)|x-y|$ and $\tilde{K}_j^L(x,y)$ is the kernel of $R_j^L=(\partial/\partial x_j)L^{-1/2}$, $j=1,\cdots,n$. In particular, when $V=0$, $K_j^\Delta(x,y)=\tilde{K}_\Delta^L(x,y)|x-y|=(|x_j-y_j|/|x-y|)/|x-y|^{n-1}$ and $\tilde{K}_\Delta^L(x,y)$ is the kernel of $R_j=(\partial/\partial x_j)\Delta^{-1/2}$, $j=1,\cdots,n$. In this paper, we write $K_j^\Delta(x,y)=K_j(x,y)$ and

$$\mu_j f(x)=\Big(\int_0^\infty\Big|\int_{|x-y|\le t}K_j(x,y)f(y)dy\Big|^2\frac{dt}{t^3}\Big)^{\frac{1}{2}}.$$

Obviously, μ_j are classical marcinkiewicz functions. Gao and Tang [4] considered the boundedness of marcinkiewicz integral μ_j^L on $L^p(\mathbb{R}^n)$. Chen and Zou [5] also proved that the marcinkiewicz integral μ_j^L has the same boundedness. The paper [6] by Tang and Dong proved the boundedness of some schrödinger type operators on Morrey spaces related to certain nonnegative potentials. Recently, Chen and Jin [7] have showed that marcinkiewicz integrals associated with Schrödinger Operator is bounded on Morrey Spaces.

It is well known that function spaces with variable exponents were intensively studied during the past 20 years, due to their applications to PDE with non-standard growth conditions and so on, we mention e.g., (see [8, 9]). A great deal of work has been done to extend the theory of maximal,potential, singular and marcinkiewicz integrals operators on the classical spaces to the variable exponent case, (see [10–14]). Recently, the author in [15] introduces a new function space that is Morrey space with variable exponents related to certain nonnegative potentials and considers the boundedness of some Schrödinger type operators on Morrey with variable exponent. Hence, it will be an interesting problem whether we can establish the boundedness of the fractional maximal operator, maximal operator and Marcinkiewicz integrals associated with Schrödinger operators on Morrey spaces with variable exponent related to certain nonnegative potentials. The main purpose of this paper is to answer the problem.

To meet the requirements in the next sections, here, basic elements of the theory of Lebsegue spaces with variable exponent are briefly presented.