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Approximation by Complex Baskakov-Szász-Durrmeyer Operators in Compact Disks

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Abstract. In this paper, we deal with the complex Baskakov-Szász-Durrmeyer mixed operators and study Voronovskaja type results with quantitative estimates for these operators attached to analytic functions of exponential growth in $\mathbb{D}_R=\{z\in\mathbb{C};|z|<R\}$. Also, the exact order of approximation is found. The method used allows to construct complex Szász-type and Baskakov-type approximation operators without involving the values on $[0,\infty)$.

Key Words: Complex Baskakov-Szász-Durrmeyer operators, Voronovskaja type result, exact order of approximation in compact disks, simultaneous approximation.

AMS Subject Classifications: 30E10, 41A25, 41A28

1 Introduction

The study of approximation properties for the Szász type operators on $[0,+\infty)$ was well established in [30] and then generalized in various ways, see e.g., [2]. Also, very recently, approximation properties for several real operators including the Szász-Durrmeyer operators are presented in the book [18]. In order to approximate integrable functions on the positive real axis, in [19] it was proposed the modifications of the Baskakov operators with the weights of Szász basis functions under the integral sign. These operators reproduce only constant functions. Ten years later in [3], it was proposed yet another sequence of the Baskakov-Szász-Durrmeyer operators which preserve constant as well as linear functions. Also, generalizations of the Durrmeyer polynomials were studied in e.g., [1].

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In the complex domain, the overconvergence phenomenon holds, that is the extension of approximation properties from real domain to complex domain. In this context, the first qualitative kind results were obtained in the papers [6,31,32]. Then, in the books [7, 8] quantitative approximation results are presented for several type of approximation operators. For Szász-Mirakjan operator and its Stancu variant in complex domain, we refer the readers to [4,5,9,21,23,24,29] and [20]. Also for complex Bernstein-Durrmeyer operators, several papers are available in the literature (see e.g., [10,11,13,14,17,25–28]), for complex Szász-Durrmeyer operators see [15], while for complex q-Balász-Szabados operators see [22].

In the present paper, we study the rate of approximation of analytic functions in a disk $\mathbb{D}_R=\{z\in\mathbb{C};|z|<R\}$, i.e., $f(z)=\sum_{k=0}^{\infty}c_kz^k$, of exponential growth, and the Voronovskaja type result, for a natural derivation from the complex operator $L_n(f)(z)$ introduced in the case of real variable in [3], and formally defined as operator of complex variable by

$$L_n(f)(z):=n\sum_{v=1}^{\infty}b_{n,v}(z)\int_0^{\infty}s_{n,v-1}(t)f(t)dt+(1+z)^{-n}f(0),\quad z\in\mathbb{C}, \tag{1.1}$$

where

$$b_{n,k}(z)=\begin{pmatrix} n+k-1\\ k\end{pmatrix}\frac{z^k}{(1+z)^{n+k}},\quad s_{n,k}(t)=e^{-nt}\frac{(nt)^k}{k!}.$$

An important relationship used for the quantitative results in approximation of an analytic function f by the complex operator $L_n(f)$ would be $L_n(f)(z)=\sum_{k=0}^{\infty}c_kL_n(e_k)(z)$, but which requires some additional hypothesis on f (because the definition of $L_n(f)(z)$ involves the values of f on $[0,+\infty)$ too) and implies restrictions on the domain of convergence. This situation can naturally be avoided, by defining directly the approximation complex operator

$$L_n^*(f)(z)=\sum_{k=0}^{\infty}c_k\cdot L_n(e_k)(z),$$

whose definition evidently that omits the values of f outside of its disk of analyticity.

In this paper we deal with the approximation properties of the complex operator $L_n^*(f)(z)$.

It is worth noting here that if instead of the above defined $L_n(f)(z)$ we consider any other Szász-type or Baskakov-type complex operator, then for $L_n^*(f)(z)$ defined as above, all the quantitative estimates in e.g., [4,5,9,12,15,16,20,21,23] hold true identically, without to need the additional hypothesis on the values of f on $[0,\infty)$ imposed there.

In the paper we denote

$$\|f\|_r=\max\{|f(z)|;\ |z|\le r\}.$$

2 Auxiliary result

In the sequel, we need the following lemma :