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Dyadic Bivariate Fourier Multipliers for Multi-Wavelets in $L^2(\mathbb{R}^2)$

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Abstract. The single 2 dilation orthogonal wavelet multipliers in one dimensional case and single A-dilation (where A is any expansive matrix with integer entries and $|detA|=2$) wavelet multipliers in high dimensional case were completely characterized by the Wutam Consortium (1998) and Z. Y. Li, et al. (2010). But there exist no more results on orthogonal multivariate wavelet matrix multipliers corresponding integer expansive dilation matrix with the absolute value of determinant not 2 in $L^2(\mathbb{R}^2)$. In this paper, we choose

$$2I_2=\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

as the dilation matrix and consider the $2I_2$-dilation orthogonal multivariate wavelet $\Psi=\{\psi_1,\psi_2,\psi_3\}$, (which is called a dyadic bivariate wavelet) multipliers. We call the 3×3 matrix-valued function $A(s)=[f_{i,j}(s)]_{3\times 3}$, where $f_{i,j}$ are measurable functions, a dyadic bivariate matrix Fourier wavelet multiplier if the inverse Fourier transform of $A(s)(\widehat{\psi_1}(s),\widehat{\psi_2}(s),\widehat{\psi_3}(s))^\top=(\widehat{g_1}(s),\widehat{g_2}(s),\widehat{g_3}(s))^\top$ is a dyadic bivariate wavelet whenever (ψ_1,ψ_2,ψ_3) is any dyadic bivariate wavelet. We give some conditions for dyadic matrix bivariate wavelet multipliers. The results extended that of Z. Y. Li and X. L. Shi (2011). As an application, we construct some useful dyadic bivariate wavelets by using dyadic Fourier matrix wavelet multipliers and use them to image denoising.

Key Words: Multi-wavelets, Fourier multipliers, image denoising.

AMS Subject Classifications: 42C15, 46C05, 47B10

1 Introduction

One natural problem in wavelet theory concerns the construction of different wavelets. Naturally, one may attempt to construct new wavelets from a known one. This approach leads to the concept of wavelet multipliers [13, 30]. In the one dimensional case,

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the single 2 dilation orthogonal Fourier wavelet multipliers have been studied extensively and characterized completely [30]. In high dimensional case, any integer expansive matrix A with $|detA|=2$ dilation single orthogonal wavelet multipliers have been characterized completely [23–25]. The Parseval multi-wavelet frame multipliers in high dimensional case and single Parseval frame wavelet multipliers have been characterized completely [26,29]. One natural question is how to characterize orthogonal multivariate wavelet multipliers corresponding integer expansive dilation matrice A with $|detA|\neq 2$. Because there are many multivariate wavelets, this question becomes very complicated. We should find an easier case to discuss this problem. We know that characterizations of MRA wavelets play an key role in discussion on single wavelet multipliers in [23–25,30]. According to [22], the number of MRA wavelets corresponding dilation matrix A in high dimensional case equals $|detA|-1$. In this paper, we consider the uniform dilation matrix

$$2I_2=\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix},$$

which is used as dilation matrix in most literature, dilation multivariate wavelet multipliers. There exist three $2I_2$-dilation MRA functions to construct a wavelet of $L^2(\mathbb{R}^2)$ (which is called a dyadic MRA bivariate wavelet by [22]). The main purpose of this paper is studying dyadic bivariate matrix Fourier wavelet multipliers in $L^2(\mathbb{R}^2)$. We will give some conditions for dyadic bivariate matrix Fourier wavelet multipliers and also give concrete examples. The results extend that in [27].

The rest of the paper is organized as follows. In the next section, we introduce notations and terms needed for this paper. Section 3 gives an example of a special dyadic MRA bivariate wavelet multiplier. In section 4, we discuss dyadic bivariate wavelet multipliers in $L^2(\mathbb{R}^2)$. Finally, we construct some useful dyadic bivariate wavelets by using multipliers and use them to image denoising.

2 Notations, definitions and preliminary results

Throughout, let

$$2I_2=\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \text{and} \quad \mathbf{s}=(s_1,s_2)\in\mathbb{R}^2.$$

Let $L^2(\mathbb{R}^2)$ be the set of all square Lebesgue integrable functions in $\mathbb{R}^2$. A $2I_2$-dilation orthogonal bivariate wavelet (or a dyadic bivariate wavelet or Parseval frame wavelet) is the function family $\Psi=\{\psi_1,\psi_2,\psi_3\}$ ($\psi_i\in L^2(\mathbb{R}^2)$, $i=1,2,3$) such that the set

$$\{2^n\psi_i(2^n\mathbf{t}-\ell): n\in\mathbb{Z}, \quad \ell\in\mathbb{Z}^2, \quad i=1,2,3\}$$

forms an orthonormal basis or Parseval frame for $L^2(\mathbb{R}^2)$. For any function $f(t)\in L^1(\mathbb{R}^2)\bigcap L^2(\mathbb{R}^2)$, its Fourier transform is defined by

$$(\mathcal{F}f)(\mathbf{s})=\widehat{f}(\mathbf{s})=\frac{1}{2\pi}\int_{\mathbb{R}^2} f(\mathbf{t})e^{-i\mathbf{t}\circ\mathbf{s}}d\mu,$$