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Weak Type Weighted Inequalities for the Commutators of the Multilinear Calderón-Zygmund Operators

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Abstract. For the commutators of multilinear Calderón-Zygmund singular integral operators with BMO functions, the weak type weighted norm inequalities with respect to $A_{\vec{p}}$ weights are obtained.

Key Words: Commutator, multilinear Calderón-Zygmund operator, multilinear maximal function, weight.

AMS Subject Classifications: 42B20, 42B25

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1 Introduction

The multilinear Calderón-Zygmund theory originated in the works of Coifman and Meyer [2,3]. Later on the topic was retaken by several authors; including Christ and Journé [1] Kenig and Stein [7], and Grafakos and Torres [5,6]. We first recall the definitions of multilinear Calderón-Zygmund singular integral operators and commutators.

Let T be a multilinear operator initially defined on the m-fold product of Schwartz spaces and taking values into the space of tempered distributions,

$$T:\mathcal{S}(\mathbf{R}^n)\times\cdots\times\mathcal{S}(\mathbf{R}^n)\longrightarrow\mathcal{S}'(\mathbf{R}^n).$$

We say that T is an multilinear Calderón-Zygmund operator if, for some $1\le q_j<\infty$, it extends to a bounded multilinear operator from $L^{q_1}\times\cdots\times L^{q_m}$ to L^q, where $1/q=1/q_1+$

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$\cdots+1/q_m$, and if there exists a function K, defined off the diagonal $x=y_1=\cdots=y_m$ in $(\mathbf{R}^n)^{m+1}$, for $\vec{f}=(f_1,\cdots,f_m)$, satisfying

$$T(\vec{f})(x)=\int_{(\mathbf{R}^n)^m}K(x,y_1,\cdots,y_m)f_1(y_1)\cdots f_m(y_m)d\vec{y}$$

for all $x\notin\bigcap_{j=1}^m \mathrm{supp} f_j$, where $d\vec{y}=dy_1\cdots dy_m$ and $\vec{y}=(y_1,\cdots,y_m)$;

$$|K(y_0,y_1,\cdots,y_m)|\le\frac{A}{(\sum_{k,j=0}^m|y_k-y_j|)^{mn}} \tag{1.1}$$

and

$$|K(y_0,\cdots,y_j,\cdots,y_m)-K(y_0,\cdots,y_j',\cdots,y_m)|\le\frac{A|y_j-y_j'|^\gamma}{(\sum_{k,j=0}^m|y_k-y_j|)^{mn+\gamma}} \tag{1.2}$$

for some $\gamma>0$ and all $0\le j\le m$, whenever $|y_j-y_j'|\le\frac{1}{2}\max_{0\le k\le m}|y_j-y_k|$.

For a vector $\vec{b}=(b_1,\cdots,b_m)$ of locally integrable functions, we define the commutator of multilinear singular integral operators

$$T_{\vec{b}}\vec{f}(x)=\sum_{j=1}^m T_{b_j}^j\vec{f}(x)=\sum_{j=1}^m\int_{\mathbb{R}^n}(b_j(x)-b_j(y_j))K(x,y_1,\cdots,y_m)f_1(y_1)\cdots f_m(y_m)d\vec{y}.$$

Recently, Lerner, Ombrosi, Pérez and Trujillo-González [8] defined a new multilinear maximal function associated to the m-linear Calderón-Zygmund operator as

$$\mathcal{M}(\vec{f})(x)=\sup_{Q\ni x}\prod_{j=1}^m\frac{1}{|Q|}\int_Q|f_j(y_j)|dy_j,$$

and developed a $A_{\vec{P}}$ weighted theory for the multilinear maximal function and multilinear Calderón-Zygmund operators.

Let $1\le p_1,\cdots,p_m<\infty$, we will write p for the number given by $1/p=1/p_1+\cdots+1/p_m$, and $\vec{P}=(p_1,\cdots,p_m)$. Given $\vec{w}=(w_1,\cdots,w_m)$, w_j are nonnegative locally integrable functions on $\mathbb{R}^n$, $j=1,\cdots,m$, set $v_{\vec{w}}=\prod_{j=1}^m w_j^{p/p_j}$. We say that $\vec{w}$ satisfies the $A_{\vec{P}}$ condition if

$$\sup_Q\Big(\frac{1}{|Q|}\int_Q v_{\vec{w}}\Big)^{1/p}\prod_{j=1}^m\Big(\frac{1}{|Q|}\int_Q w_j^{1-p_j'}\Big)^{1/p_j'}<\infty.$$

When $p_j=1$, $(\frac{1}{|Q|}\int_Q w_j^{1-p_j'})^{1/p_j'}$ is understood as $(\inf_Q w_j)^{-1}$. For $m=1$, $A_{\vec{P}}$ is the Muckenhoupt weight class A_p. We denote $A_\infty=\cup_{p>1}A_p$.

Lerner, Ombrosi, Pérez and Trujillo-González [8] obtained the following weighted inequalities for the commutators of the multilinear singular integral operators.