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Boundedness of Multilinear Oscillatory Singular Integral on Weighted Weak Hardy Spaces

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Abstract. In this paper, by using the atomic decomposition of the weighted weak Hardy space $WH^1_\omega(\mathbb{R}^n)$, the authors discuss a class of multilinear oscillatory singular integrals and obtain their boundedness from the weighted weak Hardy space $WH^1_\omega(\mathbb{R}^n)$ to the weighted weak Lebesgue space $WL^1_\omega(\mathbb{R}^n)$ for $\omega\in A_1(\mathbb{R}^n)$.

Key Words: Multilinear oscillatory singular integral, $A_1(\mathbb{R}^n)$, weighted weak Hardy space.

AMS Subject Classifications: 42B20, 42B25, 42B35

1 Introduction and main results

Recently, some people studied the boundedness of oscillatory singular integrals with polynomial phase functions. Let $P(x,y)$ be a real-valued polynomial defined on $\mathbb{R}^n\times\mathbb{R}^n$ and K be a standard Calderón-Zygmund kernel, that is, K is C^1 on $\mathbb{R}^n$ away from the origin and has mean value zero on the unit sphere centered at the origin. Define the oscillatory singular integral operator T by

$$Tf(x)=p.v.\int_{\mathbb{R}^n}e^{iP(x,y)}K(x-y)f(y)dy. \tag{1.1}$$

Ricci-Stein in [1] stated that T is bounded on $L^p(\mathbb{R}^n)$ for $1<p<\infty$. Chanillo and Christ [2] proved that T is also bounded from $L^1(\mathbb{R}^n)$ to weak $L^1(\mathbb{R}^n)$. Pan [3] introduced a variant of the Hardy space $H^1_E(\mathbb{R}^n)$, and proved that T is bounded from $H^1_E(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. For the special form $P(x,y)=P(x-y)$, Hu and Pan [4] considered the behavior of T on $H^1(\mathbb{R}^n)$.

The purpose of this paper is to study a class of multilinear operators which are closely related to the operator T defined by (1.1). Let $m\in\mathbb{N}$ and $m\geq 2$. Let Ω be homogeneous of degree zero, belong to the space $Lip_1(S^{n-1})$ and satisfy the following conditions

$$\int_{S^{n-1}}\Omega(\theta)\theta^\alpha d\theta=0 \quad \text{for} \quad \alpha\in(N\cup\{0\})^n \quad \text{and} \quad |\alpha|=m. \tag{1.2}$$

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Let A have derivatives of order m in $BMO(\mathbb{R}^n)$ and let $R_m(A;x,y)$ denote the *m-th* order Taylor series remainder of A at x about y, that is,

$$R_m(A;x,y)=A(x)-\sum_{|\alpha|\le m-1}\frac{1}{\alpha}D^\alpha A(y)(x-y)^\alpha.$$

The operator we will consider here is defined by

$$T^A f(x)=\int_{\mathbb{R}^n} e^{iP(x,y)}\frac{\Omega(x-y)}{|x-y|^{n+m}}Q_{m+1}(A;x,y)f(y)dy, \tag{1.3}$$

where

$$Q_{m+1}(A;x,y)=R_m(A;x,y)-\sum_{|\alpha|=m}\frac{1}{\alpha!}D^\alpha A(y)(x-y)^\alpha.$$

Hu and Yang [5] considered the boundedness of the operator T^A on $H^1(\mathbb{R}^n)$ when $P(x,y)=P(x-y)$. They proved that T^A is bounded from $H^1(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. In 2000, Hu , Wu and Yang [6] proved that T^A is also bounded from weighted Hardy space $H^1_\omega(\mathbb{R}^n)$ to the weighted Lebesgue space $L^1_\omega(\mathbb{R}^n)$ for $\omega\in A_1(\mathbb{R}^n)$ and from the weighted Herz-type Hardy space to the weighted Herz space.

The classical A_p weighted theory was first introduced by Muckenhoupt in [7]. And we denote the weighted measure of E by $\omega(E)$, i.e., $\omega(E)=\int_E\omega(x)dx$.

We say that $\omega\in A_1(\mathbb{R}^n)$ if

$$\frac{1}{|B|}\int_B\omega(x)dx\le C\operatorname{ess}\inf_{x\in B}\omega(x)\quad\text{for every ball}\quad B\subseteq\mathbb{R}^n.$$

And we say $\omega\in A_p(\mathbb{R}^n)$ with $1<p<\infty$, if there exists a constant $C>0$, such that

$$\Big(\frac{1}{|B|}\int_B\omega(x)dx\Big)\Big(\frac{1}{|B|}\int_B\omega(x)^{-\frac{1}{(p-1)}}dx\Big)^{p-1}\le C$$

for every ball $B\subseteq\mathbb{R}^n$.

We define $A_\infty(\mathbb{R}^n)=\bigcup\limits_{p\ge1}A_p(\mathbb{R}^n)$. So we can get

$$\varphi\in A_1(\mathbb{R}^n)\Longrightarrow\varphi\in A_\infty(\mathbb{R}^n).$$

If $\omega\in A_\infty(\mathbb{R}^n)$, we write

$$q_\omega=\inf(q>1:\omega\in A_q(\mathbb{R}^n))$$

to denote the critical index of ω.

Definition 1.1. Let ω be a weight function on $\mathbb{R}^n$, for $1\le p<\infty$, the weighted Lebesgue space is defined by

$$L^p_\omega(\mathbb{R}^n)=\Big\{f:\|f\|_{L^p_\omega}=\Big(\int_{\mathbb{R}^n}|f(x)|^p\omega(x)dx\Big)^{1/p}<\infty\Big\}.$$