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Characterizations of Null Holomorphic Sectional Curvature of *GCR*-Lightlike Submanifolds of Indefinite Nearly Kähler Manifolds

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Abstract. We obtain the expressions for sectional curvature, holomorphic sectional curvature and holomorphic bisectional curvature of a *GCR*-lightlike submanifold of an indefinite nearly Kähler manifold and obtain characterization theorems for holomorphic sectional and holomorphic bisectional curvature. We also establish a condition for a *GCR*-lightlike submanifold of an indefinite complex space form to be a null holomorphically flat.

Key Words: Indefinite nearly Kähler manifold, *GCR*-lightlike submanifold, holomorphic sectional curvature, holomorphic bisectional curvature.

AMS Subject Classifications: 53C15, 53C40, 53C50

1 Introduction

Due to the growing importance of lightlike submanifolds in mathematical physics and relativity [5] and the significant applications of *CR* structures in relativity [3, 4], Duggal and Bejancu [5] introduced the notion of *CR*-lightlike submanifolds of indefinite Kähler manifolds. Contrary to the classical theory of *CR*-submanifolds, *CR*-lightlike submanifolds do not include complex and totally real lightlike submanifolds as subcases. Therefore Duggal and Sahin [7] introduced *SCR*-lightlike submanifolds of indefinite Kähler manifold which contain complex and totally real subcases but do not include *CR* and

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SCR cases. Therefore Duggal and Sahin [8] introduced GCR-lightlike submanifolds of indefinite Kähler manifolds, which behaves as an umbrella of complex, totally real, screen real and CR-lightlike submanifolds and further studied by [11–13]. Husain and Deshmukh [10] studied CR submanifolds of nearly Kähler manifolds. Recently, Sangeet et al. [14] introduced GCR-lightlike submanifolds of indefinite nearly Kähler manifolds and obtained their existence in indefinite nearly Kähler manifolds of constant holomorphic sectional curvature c and of constant type α. In present paper, we obtain the expressions for sectional curvature, holomorphic sectional curvature and holomorphic bisectional curvature of a GCR-lightlike submanifold of an indefinite nearly Kähler manifold and obtain characterization theorems for holomorphic sectional and holomorphic bisectional curvature.

2 Lightlike submanifolds

Let $(\bar{M},\bar{g})$ be a real $(m+n)$-dimensional semi-Riemannian manifold of constant index q such that $m,n\geq 1$, $1\leq q\leq m+n-1$ and (M,g) be an m-dimensional submanifold of $\bar{M}$ and g be the induced metric of $\bar{g}$ on M. If $\bar{g}$ is degenerate on the tangent bundle TM of M then M is called a lightlike submanifold of $\bar{M}$, for detail see [5]. For a degenerate metric g on M, $TM^{\perp}$ is a degenerate n-dimensional subspace of $T_x\bar{M}$. Thus both T_xM and $T_xM^{\perp}$ are degenerate orthogonal subspaces but no longer complementary. In this case, there exists a subspace $RadT_xM=T_xM\cap T_xM^{\perp}$ which is known as radical (null) subspace. If the mapping $RadTM: x\in M\longrightarrow RadT_xM$, defines a smooth distribution on M of rank $r>0$ then the submanifold M of $\bar{M}$ is called an r-lightlike submanifold and $RadTM$ is called the radical distribution on M. Screen distribution $S(TM)$ is a semi-Riemannian complementary distribution of $Rad(TM)$ in TM therefore

$$TM=RadTM\perp S(TM) \tag{2.1}$$

and $S(TM^{\perp})$ is a complementary vector subbundle to $RadTM$ in $TM^{\perp}$. Let $tr(TM)$ and $ltr(TM)$ be complementary (but not orthogonal) vector bundles to TM in $T\bar{M}|_M$ and to $RadTM$ in $S(TM^{\perp})^{\perp}$ respectively. Then we have

$$tr(TM)=ltr(TM)\perp S(TM^{\perp}), \tag{2.2a}$$

$$T\bar{M}|_M=TM\oplus tr(TM)=(RadTM\oplus ltr(TM))\perp S(TM)\perp S(TM^{\perp}). \tag{2.2b}$$

Let u be a local coordinate neighborhood of M and consider the local quasi-orthonormal fields of frames of $\bar{M}$ along M, on u as $\{\xi_1,\cdots,\xi_r,W_{r+1},\cdots,W_n,N_1,\cdots,N_r,X_{r+1},\cdots,X_m\}$, where $\{\xi_1,\cdots,\xi_r\}$, $\{N_1,\cdots,N_r\}$ are local lightlike bases of $\Gamma(RadTM|_u)$, $\Gamma(ltr(TM)|_u)$ and $\{W_{r+1},\cdots,W_n\}$, $\{X_{r+1},\cdots,X_m\}$ are local orthonormal bases of $\Gamma(S(TM^{\perp})|_u)$ and $\Gamma(S(TM)|_u)$ respectively. For these quasi-orthonormal fields of frames, we have

Theorem 2.1 (see [5]). *Let (M,g) be an r-lightlike submanifold of a semi-Riemannian manifold $(\bar{M},\bar{g})$. Then there exists a complementary vector bundle $ltr(TM)$ of $RadTM$ in $S(TM^{\perp})^{\perp}$ and*