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Boundedness for the Singular Integral with Variable Kernel and Fractional Differentiation on Weighted Morrey Spaces

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Abstract. Let T be the singular integral operator with variable kernel, T^* be the adjoint of T and $T^\sharp$ be the pseudo-adjoint of T. Let T_1T_2 be the product of T_1 and T_2, $T_1\circ T_2$ be the pseudo product of T_1 and T_2. In this paper, we establish the boundedness for commutators of these operators and the fractional differentiation operator D^γ on the weighted Morrey spaces.

Key Words: Singular integral, variable kernel, fractional differentiation, BMO Sobolev space, weighted Morrey spaces.

AMS Subject Classifications: 42B20, 42B25

1 Introduction

Let S^{n-1} be the unit sphere in $\mathbb{R}^n$ $(n\geq 2)$ with normalized Lebesgue measure $d\sigma$. The singular integral operator with variable kernel is defined by

$$Tf(x)=\text{p.v.}\int_{\mathbb{R}^n}\frac{\Omega(x,x-y)}{|x-y|^n}f(y)dy, \tag{1.1}$$

where $\Omega(x,z)$ satisfies the following conditions:

$$\Omega(x,\lambda z)=\Omega(x,z) \quad \text{for any} \quad x,z\in\mathbb{R}^n \quad \text{and} \quad \lambda>0, \tag{1.2a}$$

$$\int_{S^{n-1}}\Omega(x,z')d\sigma(z')=0 \quad \text{for any} \quad x\in\mathbb{R}^n. \tag{1.2b}$$

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Let $m\in\mathbb{N}$, denote by $\mathcal{H}_m$ the space of surface spherical harmonics of degree m on S^{n-1} with its dimension d_m. $\{Y_{m,j}\}_{j=1}^{d_m}$ denotes the normalized complete system in $\mathcal{H}_m$. We can write (see [1,3,9])

$$\Omega(x,z')=\sum_{m\geq 0}\sum_{j=1}^{d_m}a_{m,j}(x)Y_{m,j}(z'), \tag{1.3}$$

where

$$a_{m,j}(x)=\int_{S^{n-1}}\Omega(x,z')\overline{Y_{m,j}(z')}d\sigma(z'). \tag{1.4}$$

Let

$$T_{m,j}f(x)=\frac{Y_{m,j}}{|\cdot|^n}*f(x).$$

Then we can write

$$Tf(x)=\sum_{m\geq 0}\sum_{j=1}^{d_m}a_{m,j}(x)T_{m,j}f(x). \tag{1.5}$$

Let T^* and $T^\sharp$ denote the adjoint of T and the pseudo-adjoint of T respectively, which are defined by

$$T^*f(x)=\sum_{m=0}^{\infty}\sum_{j=1}^{d_m}(-1)^m T_{m,j}(\overline{a}_{m,j}f)(x) \tag{1.6}$$

and

$$T^\sharp f(x)=\sum_{m=0}^{\infty}\sum_{j=1}^{d_m}(-1)^m \overline{a}_{m,j}(x)T_{m,j}f(x). \tag{1.7}$$

Let T_1T_2 denote the product of T_1 and T_2, $T_1\circ T_2$ denote the pseudo product of T_1 and T_2 (see [1] for the definitions).

In 1955, Calderón and Zygmund [2] investigated the L^2 boundedness of the operator T. Let D be the square root of the Laplacian operator which is defined by $\widehat{Df}(\xi)=|\xi|\widehat{f}(\xi)$. Let

$$T_1f(x)=\text{p.v.}\int_{\mathbb{R}^n}\frac{\Omega_1(x,x-y)}{|x-y|^n}f(y)dy \tag{1.8}$$

and

$$T_2f(x)=\text{p.v.}\int_{\mathbb{R}^n}\frac{\Omega_2(x,x-y)}{|x-y|^n}f(y)dy. \tag{1.9}$$