Analysis in Theory and Applications
Anal. Theory Appl., Vol. **33**, No. 3 (2017), pp. 197-205
DOI: 10.4208/ata.2017.v33.n3.1

Weighted Pseudo Almost Periodic Solutions for a Class of Hematopoiesis Model with Time-Varying Delay

Hui Zhou[1,2,*], Liu Yang[2] and Wei Jiang[3]

[1] *School of Mathematics and Statistics, Hefei Normal University, Hefei 230601, China*
[2] *School of Mathematical Science, University of Science and Technology of China, Hefei 230039, China*
[3] *School of Mathematical Science, Anhui University, Hefei 230039, China*

Received 8 October 2015; Accepted (in revised version) 15 December 2016

Abstract. In this paper, firstly, a notion of a class of generalized weighted pseudo almost periodic function is introduced, then we investigate some basic and essential properties of the space that consists of these functions. Finally, we study the existence of weighted pseudo almost periodic solutions to hematopoiesis model with time-varying delay.

Key Words: Weighted pseudo almost periodic, hematopoiesis model, time-varying delay.

AMS Subject Classifications: 34C27, 34K14, 35B15

1 Introduction

The investigation of almost periodic and pseudo almost periodic differential equations is one of the most interesting topics for many mathematicians [1–8]. In [13], Diagana introduced the concept of weighted pseudo almost periodic functions, which is a natural generalization of the concept of pseudo almost periodic functions. Since then, some interesting and important results concerning with composition theorem, translation invariance and the ergodicity of weighted pseudo almost periodic were obtained [9–14].

In recent years, the hematopoiesis models have been attracting more attentions because of their extensively realistic significance, we refer the reader to [15–20] on this topic. To de-scribe the dynamics of hematopoiesis, Mackey and Glass [15] proposed the following delay differential equation:

$$h'(t)=-\alpha h(t)+\frac{\beta}{1+h^n(t-\tau)}, \tag{1.1}$$

*Corresponding author. *Email addresses:* `zhouh16@mail.ustc.edu.cn` (H. Zhou), `yliu722@163.com` (L. Yang), `jiangwei@ahu.edu.cn` (W. Jiang)

which is the process of production of all types of blood cells generated by a remarkable self-regulated system. About the details of Eq. (1.1), we can refer [15, 16, 18]. There are some contributions on almost periodic hematopoiesis model [21–24].

Ding et al. [24] considered the following discrete hematopoiesis model:

$$\Delta u(n)=u(n+1)-u(n)=-\alpha(n)u(n)+\frac{\beta(n)}{1+u^k(n-\tau)}, \tag{1.2}$$

the authors established the existence of weighted pseudo almost periodic solutions to Eq. (1.2) by a fixed point theorem.

To our best knowledge, there are few investigations of weighted pseudo almost periodic hematopoiesis model with variable time delay. Motivated by the above discussions, the main aim of this paper is to discuss the existence of weighted pseudo almost periodic solutions for the following model of hematopoiesis with time-varying delay:

$$x'(t)=-\alpha(t)x(t)+\frac{\beta(t)}{1+x^n(t-\tau(t))}, \quad n>1. \tag{1.3}$$

Under proper assumptions, we will obtain a unique positive weighted pseudo almost periodic solution of Eq. (1.3) by using a fixed point theorem. Let us recall some notions about normal cone and a fixed point theorem.

Let **E** be a real Banach space and P be a cone in **E**. The semi-order induced by the cone P is denoted by $\leq$. That is, $x\leq y$ if and only if $y-x\in P$ for any $x,y\in P$.

A cone P of **E** is said to be normal if there exists a positive constant δ such that $\|x+y\|\geq\delta$ for any $x,y\in P,\|x\|=\|y\|=1$. In order to obtain the main results, we will state the following fixed point theorem in a cone.

Theorem 1.1 (see [25, 26])**.** *Let P be a normal cone in a real Banach space $\mathbb{X}$, and $u,v\in P$ with $u\leq v$. Suppose that $A:[u,v]\to\mathbb{X}$ satifies:*

(a) $A:[u,v]\to\mathbb{X}$ is decreasing, i.e., $Ax_1\leq Ax_2$ for all $x_1,x_2\in[u,v]$ with $x_1\geq x_2$;

(b) $A:[u,v]\to\mathbb{X}$ is a concave operator, i.e., $A[\alpha x+(1-\alpha)y]\geq\alpha Ax+(1-\alpha)Ay$ for all $x,y\in[u,v]$ and $\alpha\in[0,1]$;

(c) $Av\geq u, Au\leq\frac{1}{2}(v+Av)$. Then there exists a unique point $x^*\in[u,v]$ such that $Ax^*=x^*$.

The rest of the paper is organized as follows. In Section 2, we shall introduce some basic definitions about weighted pseudo almost periodic functions of class h, notations and lemmas. Then we will study essential properties of these functions. In Section 3, some conditions for the existence of weighted pseudo almost periodic solutions of Eq. (1.3) are established.

2 Preliminaries

Now let us state the following definitions, notations and lemmas, which will be applied to prove our main results. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^+$ is the set of nonnegative