Analysis in Theory and Applications
Anal. Theory Appl., Vol. **33**, No. 4 (2017), pp. 375-383

DOI: 10.4208/ata.2017.v33.n4.7

On a Difference Matrix and Its Properties

P. Baliarsingh and L. Nayak*

Department of Mathematics, School of Applied Sciences, KIIT University, Bhubaneswar 751024, India

Received 28 October 2016; Accepted (in revised version) 7 September 2017

Abstract. In the present paper, a new difference matrix via difference operator D is introduced. Let $x=(x_k)$ be a sequence of real numbers, then the difference operator D is defined by $D(x)_n=\sum_{k=0}^{n}(-1)^k\binom{n}{n-k}x_k$, where $n=0,1,2,3,\cdots$. Several interesting properties of the new operator D are discussed.

Key Words: Difference operators Δ^α, $B(r,s)$, $B(r,s,t,u)$, D, Cesàro operator $C(1,1)$.

AMS Subject Classifications: 46A45, 46A35, 46B45

1 Introduction, preliminaries and definitions

Let $\mathbb{R}$ and $\mathbb{N}$ be the sets of all real numbers and nonnegative integers, respectively. Let w be the space of all real valued sequences and X and Y be two subspaces of w. Then we define a matrix mapping $A:X\to Y$, as

$$(Ax)_n:=\sum_k a_{nk}x_k, \quad n\in\mathbb{N}. \tag{1.1}$$

In fact, for $x=(x_k)\in X$, Ax is called as the $A-$transform of x provided the series in (1.1) converges for each $n\in\mathbb{N}$. Moreover, the matrix $A=(a_{nk})$, $(n,k\in\mathbb{N})$ is also regarded as a linear operator. By ℓ_∞, c and c_0, we denote the spaces of all bounded, convergent and null sequences, respectively, normed by $\|x\|_\infty=\sup_k|x_k|$. Initially, Kızmaz [14] introduced the idea of difference sequence spaces associated with the spaces ℓ_∞,c and c_0 by defining the forward difference operator Δ of order one, where

$$(\Delta x)_k=x_k-x_{k+1}, \quad k\in\mathbb{N}.$$

Later on, these sequence spaces have been generalized to the case of integral order m by Et and Çolak [12] using the operator Δ^m and

$$(\Delta^m x)_k=\sum_{i=0}^{m}(-1)^i\binom{m}{i}x_{k+i}, \quad k\in\mathbb{N}.$$

*Corresponding author. *Email address:* `laxmipriyamath@gmail.com` (L. Nayak)

Recently, Baliarsingh [3] (see also [4–6, 8]) generalized the above spaces by introducing fractional difference operator Δ^{α}, where

$$(\Delta^{\alpha}x)_k=\sum_{i=0}^{\infty}(-1)^i\frac{\Gamma(\alpha+1)}{i!\Gamma(\alpha-i+1)}x_{k+i},\quad k\in\mathbb{N}.$$

In fact, for most of the cases the new operators generated on various sequence spaces can be derived from respective limiting conditions of the triangular matrix A. The summation operator S (see [11]) derived from n-th partial sum of the sequence x is defined by $S=(s_{nk})$, where

$$s_{nk}=\begin{cases}1, & k\leq n,\\ 0, & \text{otherwise.}\end{cases}$$

The well known Cesàro operator $C(1,1)$ of order one is defined by $C(1,1)=(c_{nk})$ (see [15]), where

$$c_{nk}=\begin{cases}\dfrac{1}{n+1}, & k\leq n,\\ 0, & \text{otherwise.}\end{cases}$$

The backward difference operator $\Delta^{(r)}$ of order r is defined by $\Delta^{(r)}=(\delta_{nk}^{(r)})$ (see [1]), where

$$\delta_{nk}^{(r)}=\begin{cases}(-1)^{n-k}\binom{r}{n-k}, & k\leq n,\\ 0, & \text{otherwise.}\end{cases}$$

Similarly, difference operator associated with four tuple band matrix $B(r,s,t,u)=(b_{nk})$ (see [7]) is defined by

$$b_{nk}=\begin{cases}r, & k=n,\\ s, & k=n-1,\\ t, & k=n-2,\\ u, & k=n-3,\\ 0, & \text{otherwise,}\end{cases}$$

where $r,s,t,u\in\mathbb{R}$ with the condition that $r\neq 0$. In particular, for $t=0$ and $u=0$, $B(r,s,t,u)$ reduces to the difference operator $B(r,s)$, studied by Altay and Başar [2] (see also [9]) whereas for $u=0$, it reduces to the difference operator $B(r,s,t)$, studied by Furkan et al. [13].

Let $x=(x_k)$ be a sequence in w. Now, we define the generalized difference operator D as

$$(Dx)_n=\sum_{k=0}^{n}(-1)^k\binom{n}{n-k}x_k,\quad n\in\mathbb{N}. \tag{1.2}$$