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Hopf Bifurcation of a Nonresident Computer Virus Model with Delay

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Abstract. In this paper, a delayed nonresident computer virus model with graded infection rate is considered in which the following assumption is imposed: latent computers have lower infection ability than infectious computers. With the aid of the bifurcation theory, sufficient conditions for stability of the infected equilibrium of the model and existence of the Hopf bifurcation are established. In particular, explicit formulae which determine direction and stability of the Hopf bifurcation are derived by means of the normal form theory and the center manifold reduction for functional differential equations. Finally, a numerical example is given in order to show the feasibility of the obtained theoretical findings.

Key Words: Computer virus, delay, Hopf bifurcation, SLA model, Periodic solution.

AMS Subject Classifications: 34C15, 34C23, 37G15, 37N25

1 Introduction

With the advance of software and hardware technologies, computer viruses have been a major threat to our daily life [1]. It is an important matter to understand the spread law of computer viruses over the network. To achieve this goal, many dynamical models, such as SIR model [2], SIRS model [3–5], SEIR model [6], SEIRS model [7, 8] and SEIQRS model [9, 10] have been established by scholars at home and abroad to characterize the propagation of computer viruses.

Recently, the nonresident computer viruses that do not store or execute themselves from the computer memory have caused the attentions of many researchers [11]. In order

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to analyze and protect against the nonresident computer viruses, Muroya and Kuniya proposed the following SLA computer virus model [12]:

$$\begin{cases} \dfrac{dS(t)}{dt}=b-\mu_1 S(t)-\beta_1 S(t)L(t)-\beta_2 S(t)A(t)+\gamma_1 L(t)+\gamma_2 A(t), \\ \dfrac{dL(t)}{dt}=\beta_1 S(t)L(t)+\beta_2 S(t)A(t)+\alpha_2 A(t)-(\mu_2+\alpha_1+\gamma_1)L(t), \\ \dfrac{dA(t)}{dt}=\alpha_1 L(t)-(\mu_3+\alpha_2+\gamma_2)A(t), \end{cases} \tag{1.1}$$

where $S(t)$, $L(t)$ and $A(t)$ denote the numbers of uninfected computers, latent computers and infectious computers at time t, respectively; b is the number of external computers that are accessed to the network at time t; μ_1, μ_2 and μ_3 are the rates at which the uninfected computers, latent computers and infectious computers are disconnected from the network; α_1 and α_2 are the rates of the nonresident computer viruses within latent computers are loaded into memory and nonresident computer viruses within infectious computers transfer control to the application program, respectively; β_1 and β_2 are the transmission rates of latent computers and infectious computers, respectively; γ_1 and γ_2 are the cure rates of latent computers and infectious computers, respectively. All the parameters in system (1.1) are positive constant. Muroya and Kuniya [12] investigated global stability and permanence of system (1.1).

However, studies on dynamical systems not only involve stability and permanence, but also involve some others such as bifurcation phenomenon and periodic solutions. Particularly, Hopf bifurcation of the dynamical systems with time delay are of considerable interest [13–16]. Motivated by the work above and considering that the nonresident computer viruses within latent computers need a period to be loaded into memory, we consider the following system with time delay:

$$\begin{cases} \dfrac{dS(t)}{dt}=b-\mu_1 S(t)-\beta_1 S(t)L(t)-\beta_2 S(t)A(t)+\gamma_1 L(t)+\gamma_2 A(t), \\ \dfrac{dL(t)}{dt}=\beta_1 S(t)L(t)+\beta_2 S(t)A(t)+\alpha_2 A(t)-(\mu_2+\gamma_1)L(t)-\alpha_1 L(t-\tau), \\ \dfrac{dA(t)}{dt}=\alpha_1 L(t-\tau)-(\mu_3+\alpha_2+\gamma_2)A(t), \end{cases} \tag{1.2}$$

where τ_1 is the time delay due to the period that the nonresident computer viruses within latent computers need to be loaded into memory.

The subsequent materials of this paper are organized as follows. In Section 2, stability of the infected equilibrium and existence of Hopf bifurcation are discussed by analyzing the characteristic equation of system (1.2). The formulas for determining the properties of the Hopf bifurcation are derived by using the normal form method and center manifold theory. Then, a numerical example is carried out to illustrate the validity of the theoretical results. Finally, conclusions are given in the last section.

2 Stability of the infected equilibrium and existence of Hopf bifurcation

Based on the analysis in [12] and by a direct computation, we know that if

$$R_0=\frac{b\{\beta_1+(\alpha_1\beta_2)/(\mu_3+\alpha_2+\gamma_2)\}}{\mu_1\{\mu_2+\alpha_1+\gamma_1-(\alpha_1\alpha_2)/(\mu_3+\alpha_2+\gamma_2)\}}>1,$$

then system (1.2) has a unique infected equilibrium $E_*(S_*,L_*,A_*)$, where

$$\begin{aligned}
S_*&=\frac{\mu_2+\alpha_1+\gamma_1-(\alpha_1\alpha_2)/(\mu_3+\alpha_2+\gamma_2)}{\beta_1+(\alpha_1\beta_2)/(\mu_3+\alpha_2+\gamma_2)}=\frac{b}{\mu_1R_0},\\
L_*&=\frac{b}{\mu_2+(\mu_3\alpha_1)/(\mu_3+\alpha_2+\gamma_2)}\left(1-\frac{1}{R_0}\right),\\
A_*&=\frac{b\alpha_1}{\mu_2(\mu_3+\alpha_2+\gamma_2)+\mu_3\alpha_1}\left(1-\frac{1}{R_0}\right).
\end{aligned}$$

The Jacobian matrix of system (1.2) at the infected equilibrium E_* is

$$J(E_*)=\begin{pmatrix} \lambda-a_{11} & -a_{12} & -a_{13}\\ -a_{21} & \lambda-a_{22}-b_{22}e^{-\lambda\tau} & -a_{23}\\ 0 & -b_{32}e^{-\lambda\tau} & \lambda-a_{33}\end{pmatrix},$$

where

$$\begin{aligned}
&a_{11}=-(\mu_1+\beta_1L_*+\beta_2A_*), && a_{12}=\gamma_1-\beta_1S_*, && a_{13}=\gamma_2-\beta_1S_*,\\
&a_{21}=\beta_1L_*+\beta_2A_*, && a_{22}=\beta_1S_*-(\mu_2+\gamma_1), && a_{23}=\beta_2S_*+\alpha_2,\\
&a_{33}=-(\mu_3+\alpha_2+\gamma_2), && b_{22}=-\alpha_1, && b_{32}=\alpha_1.
\end{aligned}$$

The characteristic equation is

$$\lambda^3+a_{02}\lambda^2+a_{01}\lambda+a_{00}+(b_{02}\lambda^2+b_{01}\lambda+b_{00})e^{-\lambda\tau}=0, \tag{2.1}$$

where

$$\begin{aligned}
&a_{00}=a_{33}(a_{12}a_{21}-a_{11}a_{22}), && a_{01}=a_{11}a_{22}+(a_{11}+a_{22})a_{33}-a_{12}a_{21},\\
&a_{02}=-(a_{11}+a_{22}+a_{33}), && b_{00}=b_{32}(a_{11}a_{23}-a_{13}a_{21}),\\
&b_{01}=b_{22}(a_{11}+a_{33})-a_{23}b_{32}, && b_{02}=-b_{22}.
\end{aligned}$$

When $\tau=0$, Eq. (2.1) reduces to

$$\lambda^3+(a_{02}+b_{02})\lambda^2+(a_{01}+b_{01})\lambda+a_{00}+b_{00}=0. \tag{2.2}$$

Thus, Routh-Hurwitz criterion implies that E_* is locally asymptotically stable without delay if the condition (H_1) holds

$$(H_1)\quad a_{02}+b_{02}>0,\quad (a_{02}+b_{02})(a_{01}+b_{01})>a_{00}+b_{00}>0.$$

For $\tau>0$. Substituting $\lambda=i\,\omega(\omega>0)$ into Eq. (2.1) and separating the real and imaginary parts, we can obtain

$$\begin{cases} b_{01}\omega\sin\tau\omega+(b_{00}-b_{02}\omega^2)\cos\tau\omega=a_{02}\omega^2-a_{00}, \\ b_{01}\omega\cos\tau\omega-(b_{00}-b_{02}\omega^2)\sin\tau\omega=\omega^3-a_{01}\omega. \end{cases} \tag{2.3}$$

It is easy to see from Eq. (2.3) that

$$\omega^6+a_2\omega^4+a_1\omega^2+a_0=0, \tag{2.4}$$

where

$$a_0=a_{00}^2-b_{00}^2,\quad a_1=a_{01}^2-2a_{00}a_{02}-b_{01}^2+2b_{00}b_{02},\quad a_2=a_{02}^2-2a_{01}-b_{02}^2.$$

Let $\omega^2=v$, then

$$v^3+a_2v^2+a_1v+a_0=0. \tag{2.5}$$

Define $f(v)=v^3+a_2v^2+a_1v+a_0$. Song et al. [17] obtained the following results on the distribution of roots of Eq. (2.5).

Lemma 2.1. *For the polynomial Eq.* (2.5)*,*

(1) if $a_0<0$, then Eq. (2.5) *has at least one positive roots;*

(2) if $a_0\geq 0$ and $\Delta=a_2^2-3a_1\leq 0$, then Eq. (2.5) *has no positive roots;*

(3) if $a_0\geq 0$ and $\Delta=a_2^2-3a_1>0$, then Eq. (2.5) *has positive roots if and only if $v_1^*=\frac{-a_2+\sqrt{\Delta}}{3}>0$ and $f(v_1^*)\leq 0$.*

Next, we assume that the coefficients in Eq. (2.5) *satisfy the following condition*

$$(H_2)\quad (i)\ a_0<0\ \text{ or }\ (ii)\ a_0\geq 0,\ \Delta=a_2^2-3a_1>0,\ v_1^*=\frac{-a_2+\sqrt{\Delta}}{3}>0\ \text{ and }\ f(v_1^*)\leq 0.$$

Thus, Eq. (2.4) *has at least one positive root such that Eq.* (2.1) *has a pair of purely imaginary roots $\pm i\omega_0$. The corresponding critical value τ_0 can be obtained from Eq.* (2.3)

$$\tau_0=\frac{1}{\omega_0}\arccos\frac{(b_{01}-a_{02}b_{02})\omega_0^4+(a_{02}b_{00}+a_{00}b_{02}-a_{01}b_{01})\omega_0^2-a_{00}b_{00}}{b_{01}^2\omega_0^2+(b_{00}-b_{02}\omega_0^2)^2}. \tag{2.6}$$

Taking derivative with respect to τ on both sides of Eq. (2.1), we obtain

$$\left[\frac{d\lambda}{d\tau}\right]^{-1}=-\frac{3\lambda^2+2a_{02}\lambda+a_{01}}{\lambda(\lambda^3+a_{02}\lambda^2+a_{01}\lambda+a_{00})}+\frac{2b_{02}\lambda+b_{01}}{\lambda(b_{02}\lambda^2+b_{01}\lambda+b_{00})}-\frac{\tau}{\lambda}.$$

Further, we have

$$\mathrm{Re}\left[\frac{d\lambda}{d\tau}\right]^{-1}_{\tau=\tau_0}=\frac{f'(\omega_0^2)}{(b_{00}-b_{02}\omega_0^2)^2}.$$

Thus, if the condition (H_3): $f'(\omega_0^2)\neq 0$ holds, then $\mathrm{Re}[\frac{d\lambda}{d\tau}]^{-1}_{\tau=\tau_0}\neq 0$, which implies that the transversality conditions is satisfied. From the discussions above and according to the Hopf bifurcation theorem in [18], we have the following.

Theorem 2.1. *For system* (1.2), *if the conditions* (H_1)-(H_3) *hold, then the infected equilibrium* $E_*(S_*,L_*,A_*)$ *of system* (1.2) *is locally asymptotically stable for* $\tau\in[0,\tau_0)$ *and system* (1.2) *undergoes a Hopf bifurcation at the positive equilibrium* $E_*(S_*,L_*,A_*)$ *when* $\tau=\tau_0$, *where* τ_0 *is defined in Eq.* (2.6).

3 Direction and stability of the Hopf bifurcation

Let $u_1(t)=S(t)-S_*$, $u_2(t)=L(t)-L_*$, $u_3(t)=A(t)-A_*$, $\tau=\tau_0+\mu$, $\mu\in R$. Then, we can know that $\mu=0$ is the Hopf bifurcation value for system (1.2). Rescale the time by $t\to(t/\tau)$ to normalize the time delay so that system (1.2) can be rewritten as

$$\dot{u}(t)=L_\mu u_t+F(\mu,u_t), \tag{3.1}$$

where $u_t=(u_1(t),u_2(t),u_3(t))^T\in C=C([-1,0],R^3)$,

$$L_\mu\phi=(\tau_0+\mu)(A\phi(0)+B\phi(-1)),$$

and

$$F(\mu,\phi)=(\tau_0+\mu)\begin{pmatrix}-\beta_1\phi_1(0)\phi_2(0)-\beta_2\phi_1(0)\phi_3(0)\\ \beta_1\phi_1(0)\phi_2(0)+\beta_2\phi_1(0)\phi_3(0)\\ 0\end{pmatrix},$$

where

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\0&0&a_{33}\end{pmatrix},\qquad B=\begin{pmatrix}0&0&0\\0&b_{22}&0\\0&b_{32}&0\end{pmatrix}.$$

According to the representation theorem, there exists a 3×3 matrix function with bounded variation components $\eta(\theta,\mu)$, $\theta\in[-1,0]$ such that

$$L_\mu\phi=\int_{-1}^0 d\eta(\theta,\mu)\phi(\theta),\quad \phi\in C.$$

In fact, we choose

$$\eta(\theta,\mu)=(\tau_0+\mu)(A\delta(\theta)+B\delta(\theta+1)),$$

where δ is the Dirac delta function.

For $\phi\in C([-1,0],R^3)$, define

$$A(\mu)\phi=\begin{cases}\dfrac{d\phi(\theta)}{d\theta}, & -1\le\theta<0,\\ \displaystyle\int_{-1}^{0}d\eta(\theta,\mu)\phi(\theta), & \theta=0,\end{cases}$$

and

$$R(\mu)\phi=\begin{cases}0, & -1\le\theta<0,\\ F(\mu,\phi), & \theta=0.\end{cases}$$

Then system (3.1) is equivalent to

$$\dot{u}(t)=A(\mu)u_t+R(\mu)u_t.$$

For $\varphi\in C^1([0,1]),\ (R^3)^*$, the adjoint operator A^* of A is defined as

$$A^*(\varphi)=\begin{cases}-\dfrac{d\varphi(s)}{ds}, & 0<s\le1,\\ \displaystyle\int_{-1}^{0}d\eta^T(s,0)\varphi(-s), & s=0,\end{cases}$$

and a bilinear inner product is defined by

$$\langle\varphi(s),\phi(\theta)\rangle=\bar{\varphi}(0)\phi(0)-\int_{\theta=-1}^{0}\int_{\xi=0}^{\theta}\bar{\varphi}(\xi-\theta)d\eta(\theta)\phi(\xi)d\xi, \tag{3.2}$$

where $\eta(\theta)=\eta(\theta,0)$.

Let $q(\theta)=(1,q_2,q_3)^Te^{i\omega_0\tau_0\theta}$ be the eigenvector of $A(0)$ belonging to $+i\omega_0\tau_0$ and $q^*(s)=D(1,q_2^*,q_3^*)e^{i\omega_0\tau_0 s}$ be the eigenvector of $A^*(0)$ belonging to $-i\omega_0\tau_0$. By a direct computation, we can get

$$\begin{aligned}q_2&=\frac{i\omega_0-a_{33}}{b_{32}-b_{33}e^{-i\tau_0\omega_0}}q_3,\\ q_3&=\frac{(i\omega_0-a_{11})b_{32}e^{-i\tau_0\omega_0}}{a_{12}(i\omega_0-a_{33})+a_{13}b_{32}e^{-i\tau_0\omega_0}},\\ q_2^*&=-\frac{i\omega_0+a_{11}}{a_{21}},q_3^*=\frac{a_{23}(i\omega_0+a_{11})-a_{13}a_{21}}{a_{21}(i\omega_0+a_{33})}.\end{aligned}$$

From Eq. (3.2), we can get

$$\langle q^*(s),q(\theta)\rangle=\bar{D}[1+q_2\bar{q}_2^*+q_3\bar{q}_3^*+\tau_0e^{-i\tau_0\omega_0}q_2(b_{22}\bar{q}_2^*+b_{32}\bar{q}_3^*)].$$

Then we choose

$$\bar{D}=[1+q_2\bar{q}_2^*+q_3\bar{q}_3^*+\tau_0 e^{-i\tau_0\omega_0}q_2(b_{22}\bar{q}_2^*+b_{32}\bar{q}_3^*)]^{-1},$$

such that $\langle q^*,q\rangle=1$.

Next, we can obtain the coefficients by using the method introduced in [18] and a computation process similar as that in [13]:

$$\begin{aligned}
g_{20}&=2\tau_0\bar{D}(\bar{q}_2^*-1)(\beta_1 q_2+\beta_2 q_3),\\
g_{11}&=\tau_0\bar{D}(\bar{q}_2^*-1)[\beta_1(q_2+\bar{q}_2)+\beta_2(q_3+\bar{q}_3)],\\
g_{02}&=2\beta\tau_0\bar{D}(\bar{q}_2^*-1)(\beta_1\bar{q}_2+\beta_2\bar{q}_3),\\
g_{21}&=2\beta\tau_0\bar{D}(\bar{q}_2^*-1)\Big[\beta_1\Big(W_{11}^{(1)}(0)q_2+\frac{1}{2}W_{20}^{(1)}(0)\bar{q}_2+W_{11}^{(2)}(0)+\frac{1}{2}W_{20}^{(2)}(0)\Big)\\
&\qquad+\beta_2\Big(W_{11}^{(1)}(0)q_3+\frac{1}{2}W_{20}^{(1)}(0)\bar{q}_3+W_{11}^{(3)}(0)+\frac{1}{2}W_{20}^{(3)}(0)\Big)\Big],
\end{aligned}$$

with

$$\begin{aligned}
W_{20}(\theta)&=\frac{ig_{20}q(0)}{\tau_0\omega_0}e^{i\tau_0\omega_0\theta}+\frac{i\bar{g}_{02}\bar{q}(0)}{3\tau_0\omega_0}e^{-i\tau_0\omega_0\theta}+E_1e^{2i\tau_0\omega_0\theta},\\
W_{11}(\theta)&=-\frac{ig_{11}q(0)}{\tau_0\omega_0}e^{i\tau_0\omega_0\theta}+\frac{i\bar{g}_{11}\bar{q}(0)}{\tau_0\omega_0}e^{-i\tau_0\omega_0\theta}+E_2,
\end{aligned}$$

where E_1 and E_2 are given by the following equations, respectively

$$\begin{pmatrix} 2i\omega_0-a_{11} & -a_{12} & -a_{13}\\ -a_{21} & 2i\omega_0-a_{22}-b_{22}e^{-2i\tau_0\omega_0} & -a_{23}\\ 0 & -b_{32}e^{-2i\tau_0\omega_0} & 2i\omega_0-a_{33}\end{pmatrix}E_1=\begin{pmatrix}-\beta_1q_2-\beta_2q_3\\ \beta_1q_2+\beta_2q_3\\ 0\end{pmatrix},$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22}+b_{22} & a_{23}\\ 0 & b_{32} & a_{33}\end{pmatrix}E_2=-\begin{pmatrix}-\beta_1(q_2+\bar{q}_2)-\beta_2(q_3+\bar{q}_3)\\ \beta_1(q_2+\bar{q}_2)+\beta_2(q_3+\bar{q}_3)\\ 0\end{pmatrix}.$$

Then, we can get the following coefficients which determine the properties of the Hopf bifurcation:

$$C_1(0)=\frac{i}{2\tau_0\omega_0}\Big(g_{11}g_{20}-2|g_{11}|^2-\frac{|g_{02}|^2}{3}\Big)+\frac{g_{21}}{2}, \tag{3.3a}$$

$$\mu_2=-\frac{\mathrm{Re}\{C_1(0)\}}{\mathrm{Re}\{\lambda'(\tau_0)\}}, \tag{3.3b}$$

$$\beta_2=2\mathrm{Re}\{C_1(0)\}, \tag{3.3c}$$

$$T_2=-\frac{\mathrm{Im}\{C_1(0)\}+\mu_2\mathrm{Im}\{\lambda'(\tau_0)\}}{\tau_0\omega_0}. \tag{3.3d}$$

In conclusion, we have the following results.

Theorem 3.1. *For system* (1.2)*, If $\mu_2>0$ ($\mu_2<0$), then the Hopf bifurcation is supercritical (subcritical). If $\beta_2<0$ ($\beta_2>0$), then the bifurcating periodic solutions are stable (unstable). If $T_2>0$ ($T_2<00$), then the bifurcating periodic solutions increase (decrease).*

4 Numerical simulation

In order to verify the analytical predictions obtained in our paper, we present some numerical simulations in this section. By extracting some values from [12] and taking the conditions for the existence of the Hopf bifurcation into account, we consider the following special case of system (1.2) with the parameters $b=10$, $\alpha_1=4$, $\alpha_2=1.5$, $\beta_1=1$, $\beta_2=2.5$, $\gamma_1=0.25$, $\gamma_2=0.75$, $\mu_1=1$, $\mu_2=1.5$, $\mu_3=2$. Then, we get the following system:

$$\begin{cases} \dfrac{dS(t)}{dt}=10-S(t)-S(t)L(t)-2.5S(t)A(t)+0.25L(t)+0.75A(t),\\ \dfrac{dL(t)}{dt}=S(t)L(t)+2.5S(t)A(t)+1.5A(t)-1.75L(t)-4L(t-\tau),\\ \dfrac{dA(t)}{dt}=4L(t-\tau)-4.25A(t). \end{cases} \tag{4.1}$$

By means of Matlab 7.0, we get $R_0=7.7288$ and that system (4.1) has a unique infected equilibrium $E_*(1.2939,2.5740,2.4226)$. Then, we have $a_{00}+b_{00}=163.6174$, $a_{01}+b_{01}=92.8527$, $a_{02}+b_{02}=18.3366$. Obviously, The condition (H_1) is satisfied for system (4.1). Further, we can validate that the condition (H_2) is satisfied and we can obtain $\omega_0=0.9623$, $\tau_0=1.8150$ and $f'(\omega_0^2)=152.6377>0$. That is, the condition (H_3) holds. Thus, according to Lemma 2.1, we can conclude that $E_*(1.2939,2.5740,2.4226)$ is locally asymptotically stable when $\tau\in[0,\tau_0=1.8150)$. However, when the time delay passes through τ_0, $E_*(1.2939,2.5740,2.4226)$ loses its stability and a Hopf bifurcation occurs and a family of periodic solutions bifurcate from $E_*(1.2939,2.5740,2.4226)$. The bifurcation phenomenon of system (4.1) can be illustrated by the computer simulation in Fig. 1.

5 Conclusions

In this paper, a delayed nonresident computer virus model is investigated by incorporating the time delay due to the period used to load the nonresident virus within latent computers into memory into the SLA model proposed in [12]. Compared with the conventional computer virus models such as SIRS model [3–5], the SEIRS model [7, 8] and SEIQRS model [9, 10], we not only consider the infection ability of the infective computers but also the infection ability of the latent computers. That is, the model considered in this paper is more realistic. On the other hand, the main purpose of this paper is to investigate the effect of the time delay on the model compared with the work in [12].

It is found that, under moderate conditions, the infected equilibrium of the model is locally asymptotically stable when the value of the delay is suitable small ($\tau<\tau_0$),

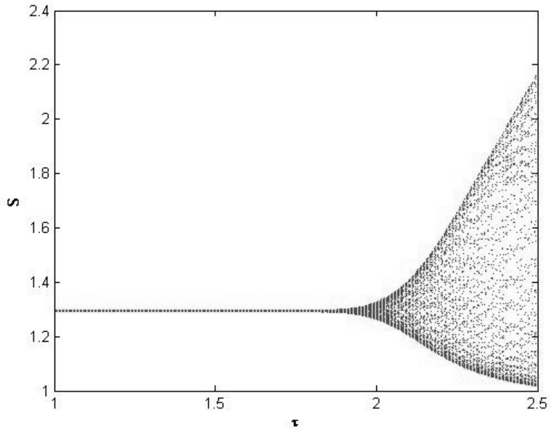

Figure 1: The bifurcation diagram with respect to τ.

which implies that propagation of the computer virus can be predicted and controlled effectively. However, a Hopf bifurcation emerges when the delay passes through the critical value τ_0. This means that the state of the computer virus prevalence changes from the infected equilibrium to a limit cycle. Namely, the propagation of the computer virus is out of control. Therefore, we can conclude that we should take some measures to postpone the occurrence of the Hopf bifurcation in order to control the propagation of the computer virus. From the numerical simulation, we find that onset of the Hopf bifurcation can be delayed if the values of the parameter γ_1 and γ_2 increase, which can be realized by means of strengthening the immunization of the new computers connected to the network. Thus, we can conclude that the managers of the network should strengthen the immunization of the new computers connected to the network so as to predict and control the propagation of the computer virus in the network easily. Furthermore, the properties of the Hopf bifurcation have also been investigated in the paper.

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