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Decomposition Formulas of Kampé de Fériets Double Hypergeometric Functions

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Abstract. This paper is sequel to the authors paper [18]. By using the generalized Burchnall-Chaundy operator method, the authors are aiming at deriving certain decomposition formulas for some interesting special cases of Kampé de Fériets series of double hypergeometric series $F^{p:q;k}_{l:m;n}$.

Key Words: Generalized hypergeometric function, Kampé de Fériets functions, decomposition formulas, inverse pairs of symbolic operators.

AMS Subject Classifications: 35K65, 33C20, 33C65

1 Introduction and preliminaries

A great interest in the theory of hypergeometric functions (that is, hypergeometric functions of one and several variables) is motivated essentially by the fact that the solutions of many applied problems involving (for example) partial differential equations are obtainable with the help of such hypergeometric functions (see, for details, [25, pp. 47-48]]. Also, in this regard, it is noticed that the general sextic equation can be solved in terms of Kampéde Fériet function (see [5]). We recall here the Kampé de Fériets function of double hypergeometric series defined by (see [2] and [24]):

$$F^{p:q;k}_{l:m;n}\left[\begin{array}{ll}(a_p):(b_q);(c_k); & \\ & x,y\\ (e_l):(f_m);(g_n); & \end{array}\right]=\sum_{r,s=0}^{\infty}\frac{((a_p))_{r+s}((b_q))_r((c_k))_s x^r y^s}{((e_l))_{r+s}((f_m))_r((g_n))_s r!s!},\tag{1.1}$$

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where for convergence

$$p+q<l+m+1,\quad p+k<l+n+1,\quad |x|<\infty,\quad |y|<\infty,$$

or

$$\begin{aligned}&p+q=l+m+1,\quad p+k=l+n+1,\\ &|x|^{\frac{1}{(p-l)}}+|y|^{\frac{1}{(p-l)}}<1,\quad \text{if } p>l;\quad \max\{|x|,|y|\}<1,\quad \text{if } p\leq l,\end{aligned}$$

and

$$((a_p))_{r+s}=\prod_{j=1}^{p}(a_j)_{r+s}=(a_1)_{r+s}(a_2)_{r+s}\cdots(a_p)_{r+s},$$

$(a)_n$ denotes the Pochhammer symbol given by $(a)_0=1$, $(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}$ and Γ is the Gamma function, with similar interpretations for $((e_r))_{r+s}$, et cetera.

Note that, the Kampé de Fériets function of double hypergeometric series (1.1) is a generalization of Appell's four double hypergeometric series F_1, F_2, F_3 and F_4 (see [2,25]).

A decomposition formula for a hypergeometric function is the one which describes the hypergeometric function with a summation of same or other hypergeometric functions. It was started to study by Burchnall and Chaundy in 1940 for Appell's double hypergeometric functions (see [4, 5]) and Chaundy [7]. Recently, it has been studying for various special functions by many mathematicians (see [1, 3, 8–22]). In particular, Hasanov and Srivastava [13,15] presented a number of decomposition formulas in terms of such simpler hypergeometric functions as the Gauss and Appell's functions and Choi-Hasanov [8] gave a formula of an analytic continuation of the Clausen hypergeometric function ${}_3F_2$ as an application of their decomposition formula. In [3], using the differential operator D and its inverse D^{-1}(defined as the integral operator and setting the lower limit to 0), Bin-Saad develops techniques to represent hypergeometric functions and their generalizations with several summation quantifiers. Here the operator (and combinations of it) is applied to simpler expressions like, e.g., products such that its image produces the desired series. In addition, new decomposition formulas are presented. Various examples illustrate how these techniques can be applied (see [3]). In [8] the authors introduced the following symbolic operators:

$$H_{x_1,x_2,\cdots,x_r}(\alpha,\beta)=\sum_{m_1,\cdots,m_r=0}^{\infty}\frac{(\beta-\alpha)_{m_1+\cdots+m_r}(-\delta_1)_{m_1}(-\delta_2)_{m_2}\cdots(-\delta_{m_r})_{m_r}}{(\beta)_{m_1+\cdots+m_r}m_1!\cdots m_r!},\tag{1.2a}$$

$$\bar{H}_{x_1,x_2,\cdots,x_r}(\alpha,\beta)=\sum_{m_1,\cdots,m_r=0}^{\infty}\frac{(\beta-\alpha)_{m_1+\cdots+m_r}(-\delta_1)_{m_1}(-\delta_2)_{m_2}\cdots(-\delta_{m_r})_{m_r}}{(1-\alpha-\delta_1-\cdots-\delta_{m_r})_{m_1+\cdots+m_r}m_1!\cdots m_r!},\tag{1.2b}$$

$$\delta_i=x_i\frac{\partial}{\partial x_i}.\tag{1.2c}$$

Based on the operators defined in (1.2a) and (1.2b), we aim in this paper to study similar type of decomposition formulas to Choi-Hasanov ones [8] for the Kampé de Fériets double hypergeoemtric series (1.1) proved by using the theory of symbolic operators.