Analysis in Theory and Applications
Anal. Theory Appl., Vol. **34**, No. 4 (2018), pp. 323-335
DOI: 10.4208/ata.OA-2017-0065

# On Chlodowsky Variant of Baskakov Type Operators

Çiğdem Atakut and Ibrahim Büyükyazıcı*

*Ankara University, Faculty of Science, Department of Mathematics,*
*06100 Tandogan, Ankara, Turkey*

Received 19 October 2017; Accepted (in revised version) 12 May 2018

**Abstract.** In the present paper, we deal with Chlodowsky type generalization of the Baskakov operators, special case of these operators includes Chlodowsky type Meyer–König and Zeller operators (see [21]). With the help of Bohman-Korovkin theorem, we obtain some approximation properties for these operators. We give a modification of the operators in the space of differentiable functions and we also present examples of graphs for approximation. Finally, we apply these operators to the solution of the differential equation.

**Key Words**: Approximation properties, Rate of convergence, Chlodowsky type MKZ operators, Baskakov operators, Differential equation.

**AMS Subject Classifications**: 41A25, 41A36, 46N20

## 1 Introduction and Construction of the Operators

Let $\{\varphi_n\}$ $(n=1,2,\cdots)$, $\varphi_n:\mathbb{C}\to\mathbb{C}$ be a sequence of functions, having the following properties:

i) $\varphi_n$ is analytic on a domain $D$ containing the disk $B=\{z\in\mathbb{C}:|z-b|\le b\}$ for each positive integer $n$,

ii) $\varphi_n(0)=1$ for $n\in\mathbb{N}$,

iii) $\varphi_n(x)>0$ and $\varphi_n^{(k)}(0)\ge 0$ for every positive integer $n$, $x\in[0,\infty)$ and and for every nonnegative integer $k$.

In [5], V. A. Baskakov introduced following the sequence of linear operators $\{L_n\}$,

$$L_n(f;x)=\frac{1}{\varphi_n(x)}\sum_{k=0}^{\infty}f\left(\frac{k}{\beta_n}\right)\varphi_n^{(k)}(0)x^k\frac{1}{k!}. \tag{1.1}$$

*Corresponding author. *Email addresses:* `atakut@science.ankara.edu.tr` (Ç. Atakut), `ibuyukyazici@gmail.com` (I. Büyükyazıcı)

Furthermore, if we take $\varphi_n(x)=(1+x)^n$ and replace $x$ by $a_n x$ in the operator (1.1), we have

$$L_n(f;x)=\frac{1}{(1+a_n x)^n}\sum_{k=0}^{n} f\Big(\frac{k}{\beta_n}\Big)\left(\begin{array}{c} n \\ k \end{array}\right)(a_n x)^k.$$

This operator is known as Bernstein type rational function which was studied in [4,16].

In [8], O. Doğru introduced and investigated approximation properties of certain linear operators defined by

$$L_n(f;x)=\frac{1}{\varphi_n(x)}\sum_{k=0}^{\infty} f\Big(\frac{k}{k+n}\Big)\varphi_n^{(k)}(0)x^k\frac{1}{k!}, \tag{1.2}$$

where for $0<b<1$ , $\varphi_n(x)$ satisfies the above conditions given by (i)-(iii) and also following condition:

iv) $\varphi_n^{(k)}(0)=\psi_n(k+n)(1+\alpha_{n,k})\varphi_n^{(k-1)}(0)$, where $\psi_n=1+\mathcal{O}(n^{-1})$ and $\alpha_{n,k}=\mathcal{O}(n^{-1})$, $\alpha_{n,k}\geq 0$, $(n,k=1,2,3,\cdots)$.

Motivated by this work, we give Chlodowsky type generalization of $L_n(f;x)$ operators given by (1.2) as follows:

Let $\{\beta_n\}_{n=1}^{\infty}$ be positive increasing sequence of real numbers with the properties

$$\lim_{n\to\infty}\beta_n=\infty \quad \text{and} \quad \lim_{n\to\infty}\frac{\beta_n}{n}=0 \tag{1.3}$$

and $\gamma$ be real number in the interval $[0,\beta_n]\subset\mathbb{R}_+=[0,\infty)$. Assume that the sequence of functions $\{\varphi_n\}$ satisfies the conditions (i)-(iv). For a function $f$ defined on $[0,\infty)$ and bounded on every finite interval $[0,\gamma]$, we define the following sequence of linear positive operators:

$$L_n(f;\beta_n,x)=\begin{cases} \dfrac{1}{\varphi_n(\frac{x}{\beta_n})}\displaystyle\sum_{k=0}^{\infty} f\Big(\frac{k}{k+n}\beta_n\Big)\varphi_n^{(k)}(0)\Big(\frac{x}{\beta_n}\Big)^k\frac{1}{k!}, & \text{if } 0\leq x<\beta_n, \\ f(x), & \text{if } x\geq\beta_n. \end{cases} \tag{1.4}$$

Recently, linear positive operators and their Chlodowsky type generalizations have been widely studied by several authors [1,3–22], because this generalization allows us to investigate approximation properties of functions defined on the infinite interval $[0,\infty)$ by using the similar techniques and methods on the classical operators.

The aim of this paper is to study some convergence properties of the operators $L_n(f;\beta_n,x)$ defined by (1.4) and modify the operators for differentiable functions, in order to improve the rate of convergence on the interval $[0,\beta_n]$ extending infinity as $n\to\infty$. Also we give an application to functional differential equation by using these operators.