Commun. Math. Anal. Appl.
doi: 10.4208/cmaa.2022-0009

Vol. **1**, No. 3, pp. 457-470
August 2022

The Fractional Ginzburg-Landau Equation with Initial Data in Morrey Spaces ϕ

Jingna Li[1,*], Xiaoting Yang[1] and Li Xia[2]

[1] *Department of Mathematics, Jinan University, Guangzhou 510632, China.*
[2] *Department of Mathematics, Guangdong University of Finance & Economics, Guangzhou 510320, China.*

Received 28 March 2022; Accepted 8 May 2022

Abstract. The paper is concerned with fractional Ginzburg-Landau equation. Existence and uniqueness of local and global mild solution with initial data in Morrey spaces are obtained by contraction mapping principle and carefully choosing the working space, further regularity of mild solution is also discussed.

AMS subject classifications: 35Q99, 35D10, 78A99

Key words: Fractional Ginzburg-Landau equation, well-posedness, Morrey space, regularity.

1 Introduction

The effects of singularities, fractal support and long-range interactions of the system are involved in numerous applications such as chaotic dynamics [18], material science [11], physical kinetics [19], and among other (see, e.g., [8, 9, 15]). Fractional dynamics equations are just the right tool to describe these phenomena because they are nonlocal, which means they depend on the value of the whole space from the mathematical point of view, see Metzler and Klafter [6].

The fractional generalization of the Ginzburg-Landau equation was first proposed by Tarasov and Zaslavsky [13]. Its rescaled form is

$$\frac{\partial u}{\partial t} = Au - (a+\nu i)\Lambda^{2\alpha} u - (b+\mu i)u|u|^{2\sigma}, \tag{1.1}$$

*Corresponding author. *Email address:* `jingna8005@hotmail.com` (J. Li)

$$u(x,0)=u_0(x), \tag{1.2}$$

where $\Lambda=(-\triangle)^{\frac{1}{2}}$, and $\sigma>0, A\geq 0, a>0, b>0, \alpha\in(0,1], \nu, \mu$ are real constants. Actually, this equation can be used to describe dynamic processes in medium with fractional mass dimension or a continuum with fractional dispersion [12]. It is indicated by asymptotic analysis that an implication of the complex fractional Ginzburg-Landau equation was the renormalization of the transition state owing to the non-locality of competition [7]. In [10], the Psi-series solution of the one-dimensional fractional Ginzburg-Landau equation was proposed and the dominant order behavior and its structure of arbitrary singular solutions are discussed. In [5], we obtained local well-posedness result for the whole space case with initial data in $L^p(\mathbb{R}^n), 1\leq p\leq\infty$, $C(\mathbb{R}^n)$ and global well-posedness result for the periodic case. In [4], we proved that the initial-value problem of (1.1)-(1.2) with $0<\sigma\leq 1$ is locally well-posed with initial data in $\dot{W}^{r,p}(\mathbb{R}^n)$ and $\dot{W}^{r,p}(\mathbb{T}^n)$, $\mathbb{T}^n=\mathbb{R}^n/Z^n$ if r and p satisfy

$$1<p<\infty, \quad \frac{\alpha}{3}<\frac{n}{p}\leq\alpha, \quad r=\frac{n}{p}-\alpha\leq 0$$

by contraction mapping principle.

As we learned from [7, 12–14, 17], very singular initial data such as certain measures concentrated on smooth surfaces are of real physical interest for fractional Ginzburg-Landau equation, which motivates us to reconsider the problem of (1.1)-(1.2) containing initial data in the Morrey space. In this paper, we prove that if $0<\alpha\leq 1$ and $u_0\in M_{p,\lambda}(\mathbb{R}^n)$ with

$$1\leq p<\infty, \quad 0\leq\lambda<\infty, \quad \frac{n-\lambda}{p}<\frac{\alpha}{\sigma},$$

then the problem of (1.1)-(1.2) is locally well-posed for some $T>0$ and for sufficiently small initial data the solution is global. Moreover, we prove that the solution is actually smooth for $0<\sigma<1$. The precise statement of the results is presented in Theorem 3.1 of Section 3. For initial data $u_0(x)\in M_{p,\lambda}(\mathbb{R}^n)$, we prove that Eq. (1.1)-(1.2) admits a solution $u\in BC([0,T);L_{-k/q,q})$, and for the global solution u we have decay rate

$$\|u\|_{L^\infty}=\mathcal{O}(t^z),$$

where

$$z=-\left[\frac{n-\lambda}{p\alpha}\left(\sigma+\frac{1}{2}\right)-1\right].$$

The results reduce to those in $L^p(\mathbb{R}^n)$ theory by taking $\lambda=0$.