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A KAM-type Theorem for Generalized Hamiltonian Systems*

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Abstract: In this paper we mainly concern the persistence of lower-dimensional invariant tori in generalized Hamiltonian systems. Here the generalized Hamiltonian systems refer to the systems which may admit a distinct number of action and angle variables. In particular, system under consideration can be odd dimensional. Under the Rüssmann type non-degenerate condition, we proved that the majority of the lower-dimension invariant tori of the integrable systems in generalized Hamiltonian system are persistent under small perturbation. The surviving lower-dimensional tori might be elliptic, hyperbolic, or of mixed type.

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1 Introduction and Result

The classical KAM theory, established by Kolmogorov[1], Arnold[2] and Moser[3] in the last century, is a landmark of the development of Hamiltonian systems. It gives a reasonable explanation for the stability of solar system and brings a new method into the study of Hamiltonian systems. The classical KAM theory established on $2n$-dimensional smoothly symplectic manifold asserts that the majority of non-resonant tori of integrable systems can survive small perturbations under the Kolmogorov non-degenerate condition. In 1967, Melnikov formulated a KAM type persistence result for lower-dimensional elliptic tori of nearly integrable Hamiltonian systems. But the complete proof was not carried out until fifteen years later it was provided by Eliasson[4], Kuksin[5], Pöschel[6]. The persistence of hyperbolic

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type lower dimensional invariant tori was treated in [7]–[12]. Parasyuk[13] foremost studied the existence of invariant tori for Hamiltonian systems with distinct numbers of action-angle variables (coisotropic).

As the generalization of the traditional Hamiltonian systems which defined on a symplectic manifold, the generalized Hamiltonian systems are defined on a Poisson manifold which can be odd dimensional and structurally degenerate. The generalized Hamiltonian systems can describe more general mathematical models, so in the study of the generalized Hamiltonian systems there are some practical meanings. The symplectic structure brings some special properties for the classical Hamiltonian systems. Since there is no symplectic structure for odd-dimensional systems, some results in classical Hamiltonian systems no longer hold. Hence the development of the KAM theory for odd-dimensional system has been considered as a challenging problem (see [14]–[16]). The theory of KAM type has been developed for volume preserving flows in [17] and [18]. For the case of diffeomorphism which is volume preserving or satisfies the intersection property, it was treated by Cheng and Sun[19], and Xia[20].

In paper [21], the authors established a KAM type theorem for the generalized Hamiltonian systems. In paper [22], the authors formulated a KAM type persistence result for lower-dimensional hyperbolic tori of nearly integrable generalized Hamiltonian systems. In this paper, we proved that the majority of the lower-dimensional invariant tori in the generalized Hamiltonian system are persistent under small perturbations, the surviving lower-dimensional tori might be elliptic, hyperbolic, or of mixed type.

Consider the Poisson manifold $(G\times T^n\times R^{2m}, \omega^2)$, where $G\subset R^l$ is a bounded, connected, closed region, T^n is the standard n-torus, and l, n, m are positive integers. The structure matrix

$$I=(A_{ij}): G\times T^n\times R^{2m}\to R^{(l+n+2m)\times(l+n+2m)}$$

associated with 2-form ω^2 is a real analytic, anti-symmetric, matrix valued function and satisfies the following two conditions:

(i) rank $I>0$;

(ii) Jacobi identity

$$\sum_{t=1}^{l+n+2m} A_{it}\frac{\partial A_{jk}}{\partial w_t}+A_{jt}\frac{\partial A_{ki}}{\partial w_t}+A_{kt}\frac{\partial A_{ij}}{\partial w_t}=0$$

holds for all $w=(y,x,z)\in G\times T^n\times R^{2m}$, $i,j,k=1,2,\cdots,l+n+2m$.

On the Poisson manifold $(G\times T^n\times R^{2m}, \omega^2)$, we consider the generalized Hamiltonian system

$$H(y,x,z)=h(y)+\frac{\delta}{2}\langle z,\ M(y)z\rangle+\varepsilon P(y,x,z), \tag{1.1}$$

where $x\in T^n$, $y\in G\subset R^l$, $z\in R^{2m}$, G is a bounded closed region, δ and ε are small parameters satisfying $\varepsilon\ll\delta$, $h(y)$, $M(y)$ and $P(y,x,z)$ are real analytic functions respectively, and $M(y)$ is a symmetric matrix.

The 2-form ω^2 is required to be invariant relative to T^n. Suppose that the unperturbed