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On Gliding Hump Properties of Matrix Domains*

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Abstract: In this note, we establish several results concerning the gliding hump properties of matrix domains. In order to discuss F-WGHP, we introduce the UAK-property and find that this sort of property has close relationship with F-WGHP. In the course of discussing F-WGHP and WGHP of $(c_0)_{C_n}$, we discuss the F-WGHP and WGHP of the almost-null sequence space f_0.

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1 Introduction

Recently, Boos and his collaborators have presented classes of infinite matrices A such that the matrix domain E_A has a certain gliding hump property whenever a given sequence space E has this property in [1]. In this note we discuss the F-WGHP and WGHP of $(c_0)_{C_n}$, then give the F-WGHP and WGHP of the almost-null sequence space f_0.

The gliding hump technique of proof was originally introduced by Lebesgue (see [2]). Now this kind of method has been used to treat numerous topics in analysis, and this kind of property was generalized extensively and used to establish some important results, and you can refer to [3], [4] for detailed information. While there are known examples of sequence spaces possessing the various gliding hump properties, there are few known examples of spaces with signed gliding hump and signed F-gliding hump properties so it would be of interest to have constructions which provide examples of sequence spaces with

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various gliding hump properties. In [1], Boos and his collaborators introduced a general procedure for constructing a sequence space from a given sequence and an infinite matrix. In this note we continue to use this procedure given by J. Boos to construct examples of sequence spaces with singed F-weak gliding hump and F-weak gliding hump properties.

2 Notations and Preliminaries

We begin by fixing the notations and describing the general procedure which we employ for generating sequence spaces from infinite matrices. Let E be a vector space of scalar sequences which contains the subspace c_{00} of all sequences which are eventually 0. Let

$$A = [a_{ij}]$$

be an infinite matrix. If $x = (x_j)$ is a scalar sequence, let

$$Ax = \Big(\sum_{j=1}^{\infty} a_{ij}x_j\Big)$$

be the image of x under the matrix A provided each series $\sum\limits_{j=1}^{\infty} a_{ij}x_j$ converge for every i. We use the sequence space E and matrix A to generate a further sequence space. We define E_A to be the vector space of all sequences x such that $Ax \in E$. Then A is a linear map from E_A into E. Some of the familiar sequence spaces can be generated by this construction. In particular, c_A and $(c_0)_A$ are the spaces of all sequences which are A-summable and A-summable to 0, respectively. Note, E_A is an FK-space whenever E is.

Example 2.1 Let $B = [b_{ij}]$ be the matrix with $b_{ij} = 1$ for $j \le i$ and $b_{ij} = 0$ otherwise. Then $l^\infty_B = bs$, the space of bounded series, and $c_B = cs$, the space of convergent series.

Example 2.2 Let n be an arbitrary nonnegative integer and $C_n = (c_{ij})$ be the matrix with $c_{ij} = 1/i$ for $n+1 \le j \le i+n$ and $c_{ij} = 0$ otherwise. Then C_n becomes the Cesàro matrix when $n = 0$. So we call C_n to be generalized Cesàro matrix. In particular, $l^\infty_{C_n}$ is the vector space of sequences with bounded averages

$$l^\infty_{C_n} = \Big\{x : \sup_{k\in N}\Big|\frac{1}{k}\sum_{j=n}^{k+n} x_j\Big| < \infty\Big\}.$$

Example 2.3 More generally, we consider Riesz matrices (means) R_p (instead C_0) also known as weighed means: we consider exclusively real sequences $p = (p_k)$ with

$$p_1 > 0, \qquad p_k \ge 0 \ \ (k \in N), \qquad \text{and} \qquad P_n := \sum_{k=1}^{n} p_k \ \ (k \in N) \tag{2.1}$$

Then the Riesz matrix $R_p = (r_{ij})$ (associated with p) is defined by

$$r_{ij} = \begin{cases} p_j/P_i, & \text{if } j \le i; \\ 0, & \text{otherwise.} \end{cases}$$

Note that if $p = (1, 1, \cdots)$, then $R_p = C_1$. Each Riesz matrix R_p is conservative and is either regular (being equivalent to $p \notin l^1$) or coercive (see [5], Section 3.2).