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The $L(3,2,1)$-labeling on Bipartite Graphs*

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Abstract: An $L(3,2,1)$-labeling of a graph G is a function from the vertex set $V(G)$ to the set of all nonnegative integers such that $|f(u)-f(v)| \geq 3$ if $d_G(u,v)=1$, $|f(u)-f(v)| \geq 2$ if $d_G(u,v)=2$, and $|f(u)-f(v)| \geq 1$ if $d_G(u,v)=3$. The $L(3,2,1)$-labeling problem is to find the smallest number $\lambda_3(G)$ such that there exists an $L(3,2,1)$-labeling function with no label greater than it. This paper studies the problem for bipartite graphs. We obtain some bounds of λ_3 for bipartite graphs and its subclasses. Moreover, we provide a best possible condition for a tree T such that $\lambda_3(T)$ attains the minimum value.

Key words: channel assignment problems, $L(2,1)$-labeling, $L(3,2,1)$-labeling, bipartite graph, tree

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1 Introduction

The problem of vertex labeling with a condition at distance two arises from the channel assignment problem introduced by Hale[1]. For a given graph G, an $L(2,1)$-labeling is defined as a function

$$f: \ V(G) \to \{0,1,2,\cdots\}$$

such that

$$|f(u)-f(v)| \geq \begin{cases} 2, & d_G(u,v)=1; \\ 1, & d_G(u,v)=2, \end{cases}$$

where $d_G(u,v)$, the distance between u and v, is the minimum length of a path between u and v. A k-$L(2,1)$-labeling is an $L(2,1)$-labeling such that no integer is greater than k. The $L(2,1)$-labeling number of G, denoted by $\lambda(G)$, is the smallest number k such that G has a

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k-$L(2,1)$-labeling. The $L(2,1)$-labeling problem has been extensively studied in recent years (see [2]–[9]).

Shao and Liu[10] extend $L(2,1)$-labeling problem to $L(3,2,1)$-labeling problem. For a given graph G, a k-$L(3,2,1)$-labeling is defined as a function

$$f: \ V(G) \to \{0,1,2,\cdots k\}$$

such that

$$|f(u)-f(v)| \geq 4-d_G(u,v), \qquad d_G(u,v) \in \{1,2,3\}.$$

The $L(3,2,1)$-labeling number of G, denoted by $\lambda_3(G)$, is the smallest number k such that G has a k-$L(3,2,1)$-labeling. Clearly,

$$\lambda_3(G) \geq 2\Delta(G)+1$$

for any non-empty graph G. It was showed that

$$\lambda_3(G) \leq \Delta^3 + 2\Delta$$

for any graph G and

$$\lambda_3(T) \leq 2\Delta + 3$$

for any tree T (see [11]). This paper focuses on bipartite graphs. In Section 2, we obtain some bounds of λ_3 for bipartite graphs and its subclasses, where the bound for bipartite graphs is $O(\Delta^2)$. In Section 3 we provide a best possible condition for a tree T with $\Delta(T) \geq 5$ and such that $\lambda_3(T)$ attains the minimum value, that is, $\lambda_3(T) = 2\Delta + 1$ if the distance between any two vertices of maximum degree is not in $\{2,4,6\}$.

All graphs considered here are non-empty, undirected, finite, simple graphs. For a graph G, we denote its vertex set, edge set and maximum degree by $V(G)$, $E(G)$ and $\Delta(G)$, respectively. For a vertex $v \in V(G)$, let

$$N_G^k(v) = \{u | d_G(u,v) = k\}, \qquad N_G[v] = N_G(v) \cup \{v\},$$

and $d_G(v)$ be the degree of v in G. A vertex of degree k is called a k-vertex. Especially, a 1-vertex of a tree is called a leaf or a pendant vertex. Let

$$D_\Delta(G) = \{d_G(u,v) | u, v \text{ are two } \Delta\text{-vertices}\}.$$

If there are no confusions in the context, we use V, Δ, λ_3, $N^k(v)$, $N[v]$, $d(v)$, $d(u,v)$ and D_Δ to denote $V(G)$, $\Delta(G)$, $\lambda_3(G)$, $N_G^k(v)$, $N_G[v]$, $d_G(v)$, $d_G(u,v)$ and $D_\Delta(G)$, respectively. And we use k-labeling to denote k-$L(3,2,1)$-labeling.

2 Bounds of λ_3 on Bipartite Graphs

First, we summarize some easy observations into the following lemma.

Lemma 2.1 *For any graph G,*

(i) *if $\lambda_3 = 2\Delta+1$ and f is a $(2\Delta+1)$-labeling, then $f(u) \in \{0, 2\Delta+1\}$ for any Δ-vertex u;*

(ii) *if f is a k-labeling of G, then $k - f$ is a k-labeling of G;*

(iii) *if G is connected and its diameter $d \in \{1,2,3\}$, then $\lambda_3 \geq (|V|-1)(4-d)$.*