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Feasibility and Stability of a Kind of Model Predictive Control with Additive Uncertainties*

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Abstract: In this paper the feasibility and stability of open-loop min-max model predictive control (OL-MMMPC) for systems with additive bounded uncertainties are considered. It is found that the OL-MMMPC may be infeasible and unstable if it is initially feasible. A sufficient condition for feasibility and stability of the OL-MMMPC is presented. Then an improved OL-MMMPC algorithm is proposed, which guarantees the robust stability of the closed-loop system once it is initially feasible. The effectiveness of this algorithm is illustrated by a simulation example.

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1 Introduction

Model predictive control (MPC) or receding horizon control enjoys a lot of industrial interest mainly due to its intrinsic ability to deal with hard constraints on the input, state and output explicitly. The nominal closed-loop stability of model predictive control has been solved systematically, but such results for the robust model predictive control with uncertainties are far from complete, and therefore have received much attention during these recent years (see [1]).

One approach to the constrained MPC for systems with uncertainties is to minimize the maximum of the objective function over the control input and the disturbance while enforcing input and state constraints for all possible disturbances. This strategy is known as min-max model predictive control (MMMPC). A typical uncertain description is bounded additive disturbances (see [2]) and the MMMPC for systems with bounded additive uncertainties

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is tackled mainly in two ways, one is open-loop MMMPC (OL-MMMPC) and the other is feedback MMMPC. There are many papers involving the analysis and optimal control of the two MMMPC problems, see, for example, [3]–[10]. Some results about feasibility and stability for the feedback MMMPC have been given in [3] and [4]. But to the authors' knowledge, no results about feasibility and stability for the OL-MMMPC have appeared in the literature. By analyzing the OL-MMMPC problem we point out that it may be infeasible and unstable if it is initially feasible, and present a sufficient condition which ensures its feasibility and stability. An improved OL-MMMPC algorithm is proposed and its stability is guaranteed.

This paper is organized as follows: In Section 2, the OL-MMMPC problem is stated and some definitions are introduced. The feasibility and stability of the OL-MMMPC are analyzed in Section 3. Additionally, a sufficient condition that ensures its feasibility and stability is also given. An improved OL-MMMPC algorithm is presented in Section 4. In Section 5, an illustrative simulation is given.

2 Problem Statement

Consider the state-space model with bounded additive uncertainties (see [2])

$$x(t+1) = Ax(t) + Bu(t) + D\theta(t) \tag{2.1}$$

with state and input constraints

$$x(t) \in X, \qquad u(t) \in U, \tag{2.2}$$

where $x(t) \in \mathbf{R}^{\dim x}$, $u(t) \in \mathbf{R}^{\dim u}$ and $\theta(t) \in \mathbf{R}^{\dim \theta}$ are state, input and uncertainty, respectively; $X \subseteq \mathbf{R}^{\dim x}$, $U \subseteq \mathbf{R}^{\dim u}$, and $\mathit{\Theta} \subset \mathbf{R}^{\dim \theta}$ are convex, compact sets and contain the origin as an interior point. It is assumed throughout the paper that

(A1) The pair (A, B) is stablizable, i.e., there exists K such that $\rho(A - BK) < 1$.

The cost function is a quadratic performance index:

$$J(\boldsymbol{\theta}, \boldsymbol{u}, x(t)) = \sum_{j=0}^{N-1} [x(t+j|t)^T Q x(t+j|t) + u(t+j|t)^T R u(t+j|t)] + x(t+N|t)^T P x(t+N|t),$$

where $x(t+j|t)$ and $u(t+j|t)$ are the prediction values of the state and input for time $t+j$ at time t, respectively. Sequences

$$\boldsymbol{u} = [u(t|t)^T, \cdots, u(t+N-1|t)^T]^T$$

and

$$\boldsymbol{\theta} = [\theta(t)^T, \cdots, \theta(t+N-1)^T]^T$$

are the values of input and uncertainty over a control horizon N, respectively.

$$\boldsymbol{\theta} \in \boldsymbol{\Theta} = \{\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, \cdots, \boldsymbol{\theta}_N^T] \in \mathbf{R}^{N\cdot\dim\theta} : \boldsymbol{\theta}_i \in \mathit{\Theta},\ i = 1, \cdots, N\}.$$

Weighting matrices Q, P are nonnegative definite and R is positive definite.

The OL-MMMPC is based on finding the control correction sequence $\boldsymbol{u}$ that minimizes $J(\boldsymbol{\theta}, \boldsymbol{u}, x)$ for the worst case of the predicted future evolution of the process state or output