Communications
in
Mathematical
Research
25(5)(2009), 385–401

The Decomposition and Uniqueness of n-Lie Superalgebras*

Wang Chun-yue[1], Zhang Qing-cheng[2] and Zhang Zhi-shang[1]

(*1. Basic Science Department, Jilin Teachers Institute of Engineering and Technology, Changchun, 130052*)

(*2. School of Mathematics and Statistics, Northeast Normal University, Changchun, 130024*)

Communicated by Liu Jian-ya

Abstract: In this paper, the definitions and some properties of n-Lie superalgebras are presented. Our main aim is to study the decomposition and uniqueness of finite dimensional n-Lie superalgebras with trivial center. Accoding to the decomposition of n-Lie superalgebras, we obtain the decomposition of inner derivation superalgebras and derivation superalgebras respectively. Furthermore, we discuss some properties about the centroid of n-Lie superalgebras, so we can see its application in the decomposition of n-Lie superalgebras.

Key words: n-Lie superalgebra, decomposition, indecomposition, derivation, centroid

2000 MR subject classification: 17B05, 17B50

Document code: A

Article ID: 1674-5647(2009)05-0385-17

1 Introduction

Filippov[1] introduced the concept of n-Lie algebra. In that paper, Filippov considered n-ary multilinear and skew symmetric operation $[x_1, \cdots, x_n]$ satisfying the generalized Jacobi identity:

$$[[x_1, \cdots, x_n], y_2, \cdots, y_n] = \sum_{i=1}^{n}[x_1, \cdots, [x_i, y_2, \cdots, y_n], \cdots, x_n]. \qquad (*)$$

n-Lie algebra originates from geometry and physics. Nambu[2] proposed the Nambu mechanics — a generalization of Hamiltonian mechanics in which the standard Poisson bracket is replaced by the ternary one. However, Nambu himself and his followers did not mention that the n-bracket is subject to the n-Jacobi identity $(*)$. This discovery was the starting point for Leno Takhtajan[3], who systematically developed the fundamental theory of n-Poisson

***Received date:** Jan. 14, 2007.

manifolds and called them Nambu-Poisson manifolds. Takhtajan[3] formulated the fundamental identity (∗) — a generalization of Jacobi identity for the Nambu bracket, which is a consistency condition for the Nambu's dynamics. Based on (∗), Takhtajan introduced the notion of Nambu Poisson manifold, which plays the same role in Nambu mechanics as that the Poission manifold plays in Hamiltonian mechanics. The linear Poisson bracket structure is equivalent to the Lie algebra structure on a dual space. The linear Nambu structures of order n are in one-to-one corresponding with Nambu-Lie algebras of order n on a dual space. We introduce the concept of n-Lie algebra as follows:

An n-Lie algebra is a vector space A over a field F on which there is defined an n-ary multilinear operation $[\,\cdot,\ \cdots,\ \cdot\,]$ satisfying the identities

$$[x_1,\ \cdots,\ x_n] = (-1)^{\tau(\sigma)}[x_{\sigma(1)}, \cdots, x_{\sigma(n)}] \qquad (\mathrm{J}_1)$$

and

$$[[x_1,\ \cdots,\ x_n],\ y_2,\ \cdots,\ y_n] = \sum_{i=1}^{n}[x_1,\ \cdots,\ [x_i,\ y_2,\ \cdots,\ y_n],\ \cdots,\ x_n], \qquad (\mathrm{J}_2)$$

where σ runs over the symmetric group S_n and the number $\tau(\sigma)$ is equal to 0 or 1 depending on the parity of the permutation σ.

The map

$$R(x_2,\ \cdots,\ x_n):\ A \longrightarrow A, \qquad R(x_2,\ \cdots,\ x_n)(x_1) = [x_1,\ \cdots,\ x_n]$$

for $x_i \in A$ is called a right multiplication defined by elements $x_2,\ \cdots,\ x_n \in A$.

A derivation of an n-Lie algebra is a linear map D of A into itself satisfying the condition

$$D[x_1,\ \cdots,\ x_n] = \sum_{i=1}^{n}[x_1,\ \cdots,\ Dx_i,\ \cdots,\ x_n] \qquad (\mathrm{J}_3)$$

for $x_i,\ \cdots,\ x_n \in A$.

By virtue of (J_2), right multiplications are derivations which are said to be inner. Let $L(A)$ denote the Lie algebra generated by all the inners of A. All the derivations of A generate a subalgebra of the Lie algebra $gl(A)$ which is called the derivation algebra of A, denoted by Der$A$.

The solvability, nilpotency and semi-simplicity of n-Lie algebra have been discussed in many papers (see [4]–[10]). On the basis of these theories, the authors propose the concept of n-Lie superalgebra, and mainly consider the decomposition and uniqueness of the finite dimensional n-Lie superalgebras with trivial center over a field F of characteristic 0. n-Lie superalgebra and Lie superalgebea are closely related, which is the same as the relation of n-Lie algebra and Lie algebra. Because of the multilinear operation, the structure on n-Lie superalgebra is different from that of Lie superalgebra, and there are many problems to be studied.

2 Preliminaries

Definition 2.1 *A Z_2-graded vector space A is called an n-Lie superalgebra over F if there is an n-ary multilinear operation $[\,\cdot,\ \cdots,\ \cdot\,]$ satisfying the following identities* (2.1), (2.2)