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Difference Equation for N-body Type Problem*

XU LE-SHUN[1,2], HAN YUE-CAI[3] AND LIU BAI-FENG[4]

(*1. College of Mathematics and Computer Science, Nanjing Normal University, Nanjing, 210097*)

(*2. Department of Applied Mathematics, Northwest A & F University, Yangling, Shanxi, 712100*)

(*3. School of Mathematics, Jilin University, Changchun, 130012*)

(*4. Department of Mathematics and Information Science, Shandong Institute of Business and Technology, Yantai, Shandong, 264005*)

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Abstract: In this paper, the difference equation for N-body type problem is established, which can be used to find the generalized solutions by computing the critical points numerically. And its validity is testified by an example from Newtonian Three-body problem with unequal masses.

Key words: Difference equation, N-bodytype problem, critical point

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1 Introduction

Since Chenciner and Montgomery[1] proved the existence of the remarkable figure-eight solution for Newtonian three-body problem in 1999, many relative studies on N-body problems have been aroused and published in many journals. In 2000, Chen[2] provided a better estimate for figure-eight solution by computing the functional of the two-body collinear solution. In 2002, Zhang and Zhou[3] offered a shorter proof of the existence for the figure-eight solution by variational method. In 2004, Ferrario and Terracini[4] provided a general condition on the symmetry groups to prove the existence of collision-free solutions by minimizing the functional. Moreover, the double choreography solutions are considered by Barutello and Terracini in [5] under the strong-force condition.

In particular, the numerical study plays an important role in finding a possible solution and proving a theoretical result with numerical assistance. In 1993, Moore[6] found some orbits numerically, in which the figure-eight orbit was included. In 2003, Kapela and Zgliczyński[7] provided a computer-assisted proof for some simple choreography solutions.

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In 2007, Roberts[8] proved the linear stability for figure-eight orbit by calculating the eigenvalues numerically. In [9], Chenciner etc. have shown the results of a numerical study of the simple choreography for Newtonian potential.

In this paper the difference equation for N-body type problem is established based on the traditional difference method but not on the Fourier series approach adopted in [9]. Then the generalized solution was found numerically by computing the critical point of the corresponding functional.

Generally, the N-body type problem is described by

$$-m_i\ddot{q}_i = \nabla U(q), \qquad i = 1, \cdots, N, \tag{N}$$

$$q(0) = q(T), \qquad \dot{q}(0) = \dot{q}(T), \tag{P}$$

where N is the number of the bodies, m_i is the mass of the ith body and q_i is the position of the ith body. $q = (q_1, \cdots, q_N) \in (\mathbf{R}^d)^N$, T is the period, and the potential function is

$$U(q) = \frac{1}{2}\sum_{1\le i\ne l\le N} U_{il}(q_l - q_i)$$

with the assumptions: for any $1 \le i \ne l \le N$ and $\xi \in \mathbf{R}^d\backslash\{0\}$, we have

$$U_{il} \in C^2(\mathbf{R}^d\backslash\{0\}, R), \qquad U_{il}(\xi) = U_{li}(\xi),$$

$$U_{il}(\xi) \to -\infty, \qquad \text{as} \quad |\xi| \to 0,$$

$$U(q_1, \cdots, q_N) \le 0, \qquad \forall\, (q_1, \cdots, q_N) \in (\mathbf{R}^d)^N,$$

where U_{il} is usually described by

$$U_{il} = -\frac{m_i m_l}{|q_i - q_l|^\alpha} \tag{1.1}$$

under the special scientific background.

2 Difference Approach

In order to formulate the difference equations for N-body type problem (N)-(P), we discretize time within one period $[0, T]$ with $k+1$ points $t_0, \cdots, t_k$ and denote the set of these discretized points as S, i.e., $S = \{t_0, \cdots, t_k\}$, and introduce a difference scheme by

$$q(t) \doteq \hat{q}(t) := \frac{t_s - t}{h}q(s-1) + \frac{t - t_{s-1}}{h}q(s), \qquad s = 1, \cdots, k+1, \tag{DS}$$

where $\hat{q}(s)$ is an abbreviation of $\hat{q}(t_s)$, t_s is the sth discretized point of $[0, T]$, $\hat{q} = (\hat{q}_1, \cdots, \hat{q}_N)$, and h is the discretized step. Moreover, the traditional forward difference scheme is used to approximate the derivative of q, which is used in [10] and the similar idea can also be found in [11]. Then the first order difference of q is

$$\Delta q(t_s) = q(s+1) - q(s),$$

and the second order difference of q is

$$\Delta^2 q(t_s) = q(s+1) - 2q(s) + q(s-1).$$

Hence the derivatives of $q(t_s)$ can be approximated by

$$\dot{q}(t_s) \doteq \frac{1}{h}\Delta q(t_s), \qquad \ddot{q}(t_s) \doteq \frac{1}{h^2}\Delta^2 q(t_s). \tag{2.1}$$