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Sub-cover-avoidance Properties and the Structure of Finite Groups*

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Abstract: A subgroup H of a group G is said to have the sub-cover-avoidance property in G if there is a chief series $1 = G_0 \leq G_1 \leq \cdots \leq G_n = G$, such that $G_{i-1}(H \cap G_i) \lhd \lhd G$ for every $i = 1, 2, \cdots, l$. In this paper, we give some characteristic conditions for a group to be solvable under the assumptions that some subgroups of a group satisfy the sub-cover-avoidance property.

Key words: sub-cover-avoidance property, maximal subgroup, Sylow subgroup, solvable group

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1 Introduction

All groups considered in this paper are finite. We use conventional notions and notations, as in [1]. G always denotes a finite group, $\pi(G)$ denotes the set of all primes dividing the order of G, and G_p is a Sylow p-subgroup of G for some $p \in \pi(G)$.

The cover-avoiding property of a subgroup was first studied by Gaschütz in [2] to study the solvable groups, later by Gillam[3] and Tomkinson[4]. In 1993, Ezquerro[5] gave some characterizations for a group G to be p-supersolvable and supersolvable under the assumption that all maximal subgroups of some Sylow subgroups of G have the cover-avoiding property in G. Recently, Guo and Shum[6] pushed further this approach and obtained some characterizations for a solvable group and a p-solvable group based on the assumption that some of its subgroups have the cover-avoiding property. More recently, Fan *et al.*[7] introduced the semi cover-avoiding property, which is the generalization not only of the cover-avoiding property but also of c-normality (see [8]).

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If M and N are normal subgroups of a group G with $N \leq M$, then the quotient group M/N is called a normal factor of G.

Definition 1.1 *Let G be a group, L/K a normal factor of G and H a subgroup of G. We say that*

i) *H covers L/K if $L \leq HK$;*

ii) *H avoids L/K if $L \cap H \leq K$;*

iii) *H has the cover-avoiding property in G, or H is a CAP subgroup of G in short, if H either covers or avoids every chief factor of G.*

Definition 1.2[7] *A subgroup H of a group G is said to have semi cover-avoiding property in G if there exists a chief series of G,*

$$1 = G_0 \leq G_1 \leq \cdots \leq G_n = G,$$

such that H either covers or avoids chief factor G_{i+1}/G_i for any $i \in \{0, 1, \cdots, n-1\}$. In this case, H is called a Semi-CAP subgroup of G, or a SCAP subgroup of G in short.

It is easy to see that a subgroup H of G either covers or avoids the chief factor M/N of G if and only if $N(H \cap M) \trianglelefteq G$. Based on this observation, we introduce the following definition.

Definition 1.3 *Let H be a subgroup of G.*

(1) *Suppose that M/N is a normal factor of G. H is said to sub-cover-avoid M/N if $N(H \cap M) \lhd \lhd G$.*

(2) *If there is a chief series $1 = G_0 \leq G_1 \leq \cdots \leq G_l = G$ of G such that H sub-cover-avoids G_i/G_{i-1} for every $i = 1, 2, \cdots, l$, then H is said to have the sub-cover-avoidance property in G. In this case, H is said a sub-CAP subgroup of G.*

From the above definitions it is obvious that a subnormal subgroup or a semi-CAP-subgroup of G is also a sub-CAP subgroup of G. But the converse is not true in general. Hence, sub-CAP subgroup is a generalization of subnormal group and semi-CAP subgroup.

Example 1.1 Suppose that G is the alternative group of degree 4. Then $\langle(123)\rangle$ is a sub-CAP subgroup, but not a subnormal subgroup, of G; whereas $\langle(12)(34)\rangle$ is a sub-CAP subgroup, but not a semi-CAP subgroup, of G. Furthermore, $\langle(1234)\rangle$ is a sub-CAP subgroup, but neither a subnormal subgroup nor a semi-CAP subgroup of G. Indeed, every subgroup of G is a sub-CAP subgroup of G (ref. Lemma 2.5).

Similar to the concept of semi-p-cover-avoiding subgroup, we give the following:

Definition 1.4 *Let H be a subgroup of G and p be a prime. If H sub-cover-avoids each p-chief factor of some chief series of G, then H is said to have the p-sub-cover-avoidance property in G. In this case, H is said a p-sub-CAP subgroup of G.*

In this paper, we obtain some new results for a finite group G to be solvable based on the assumption that some subgroups of G have the sub-cover-avoiding property (p-sub-cover-avoiding property).