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On f-edge Cover Chromatic Index of Multigraphs*

Jia Yan-bin[1,2] and Xu Chang-qing[1]
(*1. Department of Applied Mathematics, Hebei University of Technology, Tianjin, 300401*)
(*2. Group of Mathematics, Tianjin Experimental High School, Tianjin, 300074*)

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Abstract: Let G be a multigraph with vertex set $V(G)$. Assume that a positive integer $f(v)$ with $1 \le f(v) \le d(v)$ is associated with each vertex $v \in V$. An edge coloring of G is called an f-edge cover-coloring, if each color appears at each vertex v at least $f(v)$ times. Let $\chi'_{fc}(G)$ be the maximum positive integer k for which an f-edge cover-coloring with k colors of G exists. In this paper, we give a new lower bound of $\chi'_{fc}(G)$, which is sharp.

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1 Introduction

Throughout this paper, a graph $G(V, E)$ allows multiple edges but no loops, which has a finite vertex set V and a finite nonempty edge set E. Given two vertices $u, v \in V(G)$, the multiplicity $\mu(uv)$ is the number of edges jointing u and v in G. The multiplicity of v is

$$\mu(v) = \max\{\mu(uv) : u \in V\}.$$

Set

$$\mu = \max\{\mu(v) : v \in V\}.$$

When G has no multiple edges (that is $\mu = 1$), G is a simple graph. Let $\delta(G)$ denote the minimum degree of G.

An edge coloring of G is an assignment of colors to the edges of G. Associate positive integers 1, 2, $\cdots$ with colors, and call C a k-edge-coloring of G if C: $E \to \{1, 2, \cdots, k\}$. Let $i_C(v)$ denote the number of edges of G incident with vertex v that receive color i in the coloring C. For simplification, we write

$$i(v) = i_C(v)$$

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if there is no obscurity. Assume that a positive integer $f(v)$ with $1 \le f(v) \le d(v)$ is associated with each vertex $v \in V$. C is called an f-edge cover-coloring of G, if for each vertex $v \in V$,

$$i_C(v) \ge f(v), \qquad i = 1, 2, \cdots, k.$$

Let $\chi'_{fc}(G)$ denote the maximum positive integer k for which an f-edge cover-coloring with k colors of G exists. $\chi'_{fc}(G)$ is called the f-edge cover-coloring chromatic index of G. If

$$f(v) = 1 \qquad \text{for all } v \in V,$$

then the f-edge cover-coloring problem is reduced to edge-cover problem which had been studied in [1], [2] and [3]. The edge cover chromatic index of G is denoted by $\chi'_c(G)$. When G is a simple graph, Gupta[1] proved that

$$\delta(G) - 1 \le \chi'_c(G) \le \delta(G).$$

When G is a multigraph, Gupta gave the following famous result in [1], which can also be obtained from the result in [4].

Theorem 1.1[1] *For any graph G,*

$$\min\{d(v) - \mu(v) : v \in V\} \le \chi'_c(G) \le \delta(G).$$

Xu and Liu[3] improved the lower bound when $2 \le \delta(G) \le 5$.

Theorem 1.2[3] *For any graph G with $2 \le \delta(G) \le 5$,*

$$\chi'_c(G) \ge \delta(G) - 1.$$

How about the general lower bound? Does the general lower bound can be improved? Alon *et al.*[5] obtained the following result.

Theorem 1.3[5] *For any graph G,*

$$\chi'_c(G) \ge \left\lfloor \frac{3\delta(G) + 1}{4} \right\rfloor.$$

It can be easily proved that this lower bound is sharp. Motivated by this, we give a new lower bound for $\chi'_{fc}(G)$.

2 The Lower Bound for $\chi'_{fc}(G)$

Song and Liu[4] gave a general lower bound for $\chi'_{fc}(G)$.

Theorem 2.1 *For any graph G, let*

$$1 \le f(v) \le d(v), \qquad v \in V(G).$$

Then

$$\chi'_{fc}(G) \ge \min\left\{ \left\lfloor \frac{d(v) - \mu(v)}{f(v)} \right\rfloor, \ v \in V(G) \right\}.$$

Using the similar method in [5], we get a new lower bound for $\chi'_{fc}(G)$.