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Surface Reconstruction of 3D Scattered Data with Radial Basis Functions*

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Abstract: We use Radial Basis Functions (RBFs) to reconstruct smooth surfaces from 3D scattered data. An object's surface is defined implicitly as the zero set of an RBF fitted to the given surface data. We propose improvements on the methods of surface reconstruction with radial basis functions. A sparse approximation set of scattered data is constructed by reducing the number of interpolating points on the surface. We present an adaptive method for finding the off-surface normal points. The order of the equation decreases greatly as the number of the off-surface constraints reduces gradually. Experimental results are provided to illustrate that the proposed method is robust and may draw beautiful graphics.
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1 Introduction

The problem of reconstructing a surface from a large set of scattered data arises in a variety of applications in the fields of reverse engineering, computer graphics, computer vision, medical image segmentation, etc. Smooth and deformable surfaces, especially those enclosing a volume, are usually difficult and inefficient to model and represent by using traditional parametric surfaces. An increasingly popular modeling approach in recent years is the field-based implicit surfaces (see [1]–[3]). The implicit representations are convenient for modeling and animating complex smooth objects, such as liquids, clouds.

Implicit surfaces based on radial basis functions, which are relatively new in implicit surface modeling, have recently attracted attention because they can easily represent com-

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plex shapes with arbitrary topology. About 20 years ago, Franke[4] identified radial basis functions as one of the most accurate and stable method to solve scattered data interpolation problems. Savchenko *et al.*[5], Carr *et al.*[6], and Turk and O'Brien[7] used globally supported raial basis functions to reconstruct smooth surfaces from scattered data. Unfortunately, the RBFs have golbal support, and the equations lead to a dense linear system. Hence, these techniques fail to reconstruct surfaces from large point sets. Morse *et al.*[8], Kojekine *et al.*[9] empolied compactly supported RBFs to recontruct smooth surfaces from scattered data, but the radius of support has to be choosen globally, which means that the method is not robust against non-uniform datasets. Ohtake *et al.*[10] proposed a multi-scale approach that overcomes this limitation, but it is not feasible for the approximation of noisy data.

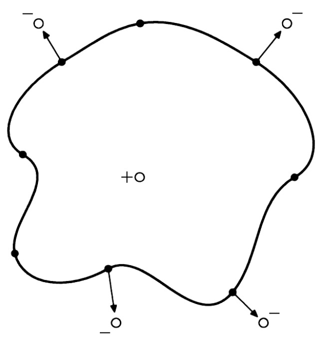
Globally supported RBFs are extremely useful in reconstructing non-uniform datasets and repairing incomplete data (see [6]), However, this work was restricted to small problems by the $O(N^2)$ storage and $O(N^3)$ arithmetic operations of direct methods. This paper propose an efficient fitting and fast evaluation method to 3D scattered data interpolation with globally supported RBFs. Firstly, a sparse approximation set of scattered data is constructed by reducing the number of interpolating points on the surface. Secondly, we present an adaptive method for finding the off-surface normal points. Finally, the order of the equation decreases greatly as the number of the off-surface constraints reduces gradually.

2 Problem Statement and Notations

The surface representation or reconstruction problem can be expressed as:

Problem 2.1 Given N distinct points $X = \{x_i\}_{i=1}^{N}$ on a surface M in $\mathbf{R}^3$, find a surface M' that is a reasonable approximation to M.

Note that we use the notation $x = (x, y, z)$ for points $x \in \mathbf{R}^3$. The approach in [6] is to model the surface implicitly with a function $f(x)$. If a surface M consists of all the points x that satisfy the equation $f(x) = 0$, then we say that f implicitly defines M.



• curve points, $f(x) = 0$;
○ off-surface normal points, $f(x)$ has negative values outside the curve and positive values inside tthe curve;
→ normal direction.

Fig. 2.1 Curve defined using interpolating implicit function