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Upper Large Deviations for Mixing Random Sequence*

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Abstract: In this article, we prove upper large deviations for the empirical measure generated by stationary mixing random sequence under some suitable assumptions and upper large deviations for the mixing random sequence.

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1 Introduction

We say that a family of probability measures $\{\mu_\varepsilon\}$ on a topological space $\mathcal{X}$ equipped with a σ-field $\mathcal{B}$ satisfies the Large Deviations Principle (LDP), if there is a lower semicontinuous rate function $I : \mathcal{X} \to [0, +\infty]$, with compact level sets $I^{-1}([0, a])$ for all $a > 0$, such that for all $\Gamma \in \mathcal{B}$,

$$-\inf_{x\in\Gamma^o} I(x) \le \liminf_{\varepsilon\to 0} \varepsilon \lg \mu_\varepsilon(\Gamma) \le \limsup_{\varepsilon\to 0} \varepsilon \lg \mu_\varepsilon(\Gamma) \le -\inf_{x\in\bar{\Gamma}} I(x),$$

where $\bar{\Gamma}$ denotes the closure of Γ, Γ^o the interior of Γ, and the infimum of a function over an empty set is interpreted as ∞. The left-hand inequality is called the Lower Large Deviations (LLD) and the right-hand inequality is called the Upper Large Deviations (ULD). We say that $\{\mu_\varepsilon\}$ satisfies the weak star Upper Large Deviations Principle (w*-ULD) with a rate function I, if for any compact subset K of $\mathcal{X}$ belonging to $\mathcal{B}$ and for any $\delta > 0$, there exists an open set G^δ of K, such that

$$\limsup_{\varepsilon\to 0} \varepsilon \lg \mu_\varepsilon \left(G^\delta\right) \le -\inf_x \min\{I(x) - \delta, 1/\delta\}.$$

Note that $\mathcal{B}$ need not necessarily be $\mathcal{B}_\mathcal{X}$ (the Borel σ-field of $\mathcal{X}$).

The LDP, which was studied in [1]–[4] and so on, has been applied to many fields such as statistical mechanics, stochastic differential equations, dynamic systems and statistics, etc. However, in comparison to other limit theorems, few papers deal with strong mixing

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dependence conditions. In this paper we prove the ULD for mixing stationary random sequence with fast enough convergence rate. Our results are motivated by [5], where a weaker conclusion was proved under a much weaker mixing assumption.

Let $\{X_i, i \in \mathbf{N}\}$ be a stationary sequence. Define σ-fields $\mathcal{F}_{a,b} = \sigma(X_i : a \le i \le b)$, and for $n \in \mathbf{N}$ define

$$\psi_+(n) = \sup\left\{\frac{P(A\cap B)}{P(A)P(B)} :\ A \in \mathcal{F}_{1,l},\ B \in \mathcal{F}_{l+n,+\infty},\ P(A)P(B) > 0,\ l \in \mathbf{N}\right\}, \tag{1.1}$$

$$\psi_-(n) = \inf\left\{\frac{P(A\cap B)}{P(A)P(B)} :\ A \in \mathcal{F}_{1,l},\ B \in \mathcal{F}_{l+n,+\infty},\ P(A)P(B) > 0,\ l \in \mathbf{N}\right\}, \tag{1.2}$$

$$\psi(n) = \frac{\psi_+(n)}{\psi_-(n)}, \tag{1.3}$$

$$\phi(n) = \sup\{P(B|A) - P(B) :\ A \in \mathcal{F}_{1,l},\ B \in \mathcal{F}_{l+n,+\infty}, P(A) > 0,\ l \in \mathbf{N}\}. \tag{1.4}$$

We say that $\{X_i, i \in \mathbf{N}\}$ is ϕ-mixing, if $\phi(n) \to 0$, as $n \to \infty$.

Our paper is organized as follows:

In Section 2 we prove the ULD for the empirical measure generated by the mixing random sequence under some suitable conditions. In Section 3 we prove the ULD for the mixing random sequence under some suitable conditions.

Throughout this paper, let E be a Polish space (i.e., a complete separable metric space), $C_b(E)$ denote all the bounded continuous functions on E, and $M_1(E)$ denote all probability measures on E, B is a separable Banach Space with the norm $\|\cdot\|$, and B^* is the topological dual of B, C denotes a positive constant, whose value can differ in different place.

To close this introduction, we remark that we have not obtained the LLD in the framework of Sections 2 and 3 untill now. We will do further research on the LLD in future.

2 ULD for Empirical Measure Sequence

Let $\{X_n, n \ge 1\}$ be a stationary sequence of random variables which take values in a Polish space E. Consider the empirical measures

$$L_n = \frac{1}{n}\sum_{i=1}^{n} \delta_{X_i}, \qquad n \ge 1,$$

where δ_x denotes the probability measure degenerate at $y \in E$. Let a space $M_1(E)$ with weak convergence topology "$\xrightarrow{w}$" and Borel σ-field $\mathcal{B}(M_1(E))$ be given. Then L_n is a random element on $M_1(E)$. Sanov's well known theorem states that μ_n satisfies the LDP when $\{X_n\}$ are i.i.d. random variables. This result has been extended to various directions. In one direction, this LDP has been shown to hold for ϕ-mixing stationary process in compact spaces (see [5]).

In this section we prove two general results on ULD for empirical measure generated by mixing random sequence. Theorem 2.1 extends the ULD for empirical measure to dependent case. Theorem 2.2 weakens the conditions of Theorem 3 (see [5]), and rules out the restriction of compact space.