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A Class of Left E-adequate Semigroups*

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Abstract: In this paper we establish a construction of a class of left E-adequate semigroups by using semilattices of cancellative monoids and fundamental left E-adequate semigroups. We first introduce concepts of type μ^+ (μ^*, μ) abundant semigroups and type μ^+ left E-adequate semigroups. In fact, regular semigroups are type μ^+ abundant semigroups and inverse semigroups are type μ^+ left E-adequate semigroups. Next, we construct a special kind of algebras called E^+-product. It is proved that every E^+-product is a type μ^+ left E-adequate semigroup, and every type μ^+ left E-adequate semigroup is isomorphic to an E^+-product of a semilattice of cancellative monoids with a fundamental left E-adequate semigroup. Finally, as a corollary of the main result, it is deduced that every inverse semigroup is isomorphic to an E^+-product of a Clifford semigroup by a fundamental inverse semigroup.

Key words: type μ^+ semigroup, abundant semigroup, left E-adequate semigroup, E^+-product

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1 Introduction

El-Qallali *et al.*[1] presented a Munn type representation for a class of E-semiadequate semigroups. As special cases of E-semiadequate semigroups they introduced three classes of abundant semigroups: left E-adequate semigroups, right E-adequate semigroups and E-adequate semigroups which are both left E-adequate and right E-adequate. The aim of this paper is to establish a construction of a class of left E-adequate semigroups, showing that the "building bricks" in this construction are semilattices of cancellative monoids and fundamental left E-adequate semigroups.

In Section 2, we first recall some known results of abundant semigroups, and introduce

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concepts of type μ^+, μ^* and μ abundant semigroups. Next we define type μ^+ left E-adequate semigroups. In fact, a regular semigroups is a type μ abundant semigroup but not a type μ^+ left E-adequate semigroup in general. In particular, an inverse semigroup is a type μ and also a type μ^+ left E-adequate semigroup. We give an example which is a type μ^+ left E-adequate semigroup, but neither type μ^* and nor right E-adequate. In Section 3, we first use semilattices of cancellative monoids and fundamental left E-adequate semigroups to construct a class of algebras, E^+-products. Next, we prove that every E^+-product is a type μ^+ left E-adequate semigroup. Section 4 shows that every type μ^+ left E-adequate semigroup can be constructed in this way. Finally, as a corollary of the main theorem we obtain that every inverse semigroup is isomorphic to a Y-product of a Clifford semigroup and a fundamental inverse semigroup.

In this paper, for the undefined notion and notations the reader is referred to [1]–[7].

2 Preliminaries

We first recall some basic facts about the equivalence relations $\mathcal{L}^*$ and $\mathcal{R}^*$ (or denoted by $\mathcal{L}^*(S)$ and $\mathcal{R}^*(S)$ respectively in case of ambiguity) on a semigroup (see [8], [9]). For elements a, b of a semigroup S, $a\mathcal{L}^*b$ means that

$$ax = ay$$

if and only if

$$bx = by, \qquad x, y \in S^1.$$

The relation $\mathcal{R}^*$ is defined dually. Evidently, $\mathcal{L}^*$ is a right congruence and $\mathcal{R}^*$ is a left congruence on S. For any result about $\mathcal{L}^*$ there exists a dual result for $\mathcal{R}^*$. In particular, we emphasize that an idempotent is a right (left) identity for its $\mathcal{L}^*$-class ($\mathcal{R}^*$-class). We define

$$\mathcal{H}^* = \mathcal{L}^* \cap \mathcal{R}^*.$$

The $\mathcal{L}^*$-class ($\mathcal{R}^*$-class, $\mathcal{H}^*$-class) containing the element a of the semigroup S is denoted by L^*_a (R^*_a) or by $L^*_a(S)$ ($R^*_a(S)$) in case of ambiguity. A semigroup is called abundant if each $\mathcal{L}^*$-class and each $\mathcal{R}^*$-class contain an idempotent (see [5]). As in [2], for $a \in S$, a^* denotes a typical idempotent in $L^*_a(S) \cap E(S)$ and a^+ denotes a typical element in $R^*_a(S) \cap E(S)$. Let E be a semilattice and a subsemigroup of an abundant semigroup S. We say that S is left (right) E-adequate (see [1]) if every $\mathcal{R}^*$-class ($\mathcal{L}^*$-class) of S contains an idempotent of E. If S is left and right E-adequate, then S is E-adequate. Further, if S is E-adequate and $E = E(S)$, then we say that S is adequate (see [5]). If S is a left (right) E-adequate semigroup, the notation E denotes a fixed semilattice in S such that $R^*_a \cap E \neq \emptyset$ ($L^*_a \cap E \neq \emptyset$) for all $a \in S$. Let X be a non-empty set, the notation $\mathcal{T}(X)$ (see [7]) denotes the full transformation semigroup of X. The notation ε_X denotes the identity transformation on X.

Lemma 2.1 ([6], Corollary 1.2) *Let a be an element of a semigroup S and e be an idempotent of S. Then the following conditions are equivalent*: