Communications in Mathematical Research
27(1)(2011), 1–5

Upper and Lower Semicontinuity of Solution Sets for Parametric Generalized Vector Quasi-equilibrium Problems*

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Communicated by Ji You-qing

Abstract: In this paper, two kinds of parametric generalized vector quasi-equilibrium problems are introduced and the relations between them are studied. The upper and lower semicontinuity of their solution sets to parameters are investigated.
Key words: parametric generalized vector quasi-equilibrium problem, solution set, upper semicontinuity, lower semicontinuity, set-valued mapping
2000 MR subject classification: 90C30
Document code: A
Article ID: 1674-5647(2011)01-0001-05

1 Introduction and Preliminaries

Equilibrium theory, including optimization problems, variational inequality problems, saddle point problems and complementary problems as special cases, provides us a general framework for studying other fields. Up to now, main efforts for equilibrium problems have been made for the solution existence; see, e.g., [1]–[3], and the references therein. A few results have been obtained for properties of solution sets; see, e.g., [4]–[6], in which stability of solutions to parameters were studied. In most cases, stability can be viewed as the semicontinuity, continuity, Lipschitz continuity or some kinds of (generalized) differentiability of solutions to parameters. Although much efforts have been made for establishing the continuity, Lipschitz continuity and (generalized) differentiability of solutions to parameters, few works was concentrated on the semicontinuity of the solution sets.

Khanh and Luu[5] studied the lower and upper semicontinuity of the solution sets for multivalued quasivariational inequalities with a single parameter. Anh and Khanh[6] considered the semicontinuity of solution sets for parametric multivalued vector quasiequilibrium problems. Inspired and motivated by their works, in this paper, we introduce two kinds

*Received date: Dec. 9, 2008.
Foundation item: The NSF (10871226) of China and the NSF (ZR2009AL006) of Shandong Province.

of parametric generalized vector quasi-equilibrium problems, which are more general than that in the literature, and study the upper and lower semicontinuity of the solution sets to parameters under some relaxed assumptions.

Throughout this paper, let X, Y, Z, Λ, M be real Hausdorff topological vector spaces, D be a nonempty compact subset of X and E a nonempty subset of Y. Let 2^E denote the family of all nonempty subsets of E, and $T : D\times\Lambda \to 2^E$, $G : D\times M \to 2^D$ and $C : D \to 2^Z$ be set-valued mappings such that $C(x)$ is a closed convex pointed cone in Z and $\text{int}C(x) \neq \emptyset$ for each $x \in D$. Let $f : D \times E \times D \to Z$ be a single-valued mapping.

A set-valued mapping $F : X \to 2^Y$ is said to be upper semicontinuous (shortly, u.s.c.) at $x_0 \in X$ if for any open set $V \supseteq F(x_0)$, there exists an open neighborhood U of x_0 such that $F(U) \subseteq V$. F is said to be u.s.c. on X if it is u.s.c. at each point in X.

$F : X \to 2^Y$ is said to be lower semicontinuous (shortly, l.s.c.) at $x_0 \in X$ if for each $y \in F(x_0)$ and any open neighborhood V of y, there exists an open neighborhood U of x_0 such that $F(z) \cap V \neq \emptyset$ for each $z \in U$, which can be equivalently stated as: F is said to be l.s.c. at x_0 if for any net $\{x_\alpha\}$ with $x_\alpha \to x_0$ and any $y \in F(x_0)$, there exists a net $\{y_\alpha\}$ with $y_\alpha \in F(x_\alpha)$ for all α such that $y_\alpha \to y$. F is said to be l.s.c. on X if it is l.s.c. at each point in X.

$F : X \to 2^Y$ is said to be a closed set-valued mapping if its graph, denoted by graph(F), is a closed set in $X \times Y$, where $\text{graph}(F) = \{(x, y) : x \in X,\ y \in F(x)\}$.

A single-valued mapping $f : D\times E\times D \to Z$ is said to be $Y\setminus -\text{int}C(x)$-quasiconvex with respect to T of type $\varPi$ if for any nonempty finite subset $\{z_1, \cdots, z_n\} \subseteq D$, any $x \in \text{co}\{z_1, \cdots, z_n\}$ and any $\lambda \in \Lambda$, there exists some i $(i = 1, \cdots, n)$ and $y \in T(x, \lambda)$ such that $f(x, y, z_i) \in Y\setminus(-\text{int}C(x))$.

2 Parametric Generalized Vector Quasi-equilibrium Problems

In this section, we introduce two kinds of parametric generalized vector quasi-equilibrium problems (shortly, PGVQEP) and study the relations between them.

For any given parameters $\lambda \in \Lambda$ and $\mu \in M$, we consider the following two parametric generalized vector quasi-equilibrium problems:

(PGVQEP1) Find $x \in D$ such that there exists $y \in T(x, \lambda)$ satisfying

$$f(x, y, z) \notin -\text{int}C(x), \qquad z \in G(x, \mu).$$

(PGVQEP2) Find $x \in D$ such that for any $z \in G(x, \mu)$ there exists $y \in T(x, \lambda)$ satisfying

$$f(x, y, z) \notin -\text{int}C(x).$$

We denote their solution sets by $S_1(\lambda, \mu)$ and $S_2(\lambda, \mu)$, respectively. Firstly, we study the nonempty of the solution set $S_1(\lambda, \mu)$.

Theorem 2.1 *For (PGVQEP1), let*

(i) *the set $\{x \in D : z \in G(x, \mu)\}$ be open for any $z \in D$ and $\mu \in M$;*

(ii) *f be $Y\setminus -\text{int}C(x)$-quasiconvex with respect to T of type $\varPi$;*