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Superderivations for a Family of Lie Superalgebras of Special Type*

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Abstract: By means of generators, superderivations are completely determined for a family of Lie superalgebras of Special type, the tensor products of the exterior algebras and the finite-dimensional Special Lie algebras over a field of characteristic $p > 3$. In particular, the structure of the outer superderivation algebra is concretely formulated and the dimension of the first cohomology group is given.

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1 Introduction

The four families of finite-dimensional simple modular Lie superalgebras of Cartan type were constructed and studied by Zhang[1] in 1997. Now people have obtained many useful results relative to structures and representations of modular Lie superalgebras (see, for example, [2]–[5]). Determining the (super)derivation algebra for a modular Lie (super)algebra is of particular interest, since a centerless Lie (super)algebra, in general, can be embedded into its (super)derivation algebra, which possesses a natural p- or $(p, 2p)$-structure. As is well-known, Lie (super)algebras with such a structure are more manageable and more interesting than the usual ones. Moreover, the p-envelope contained in the (super)derivation algebra can be easily computed. Certain work on the superderivations of modular Lie superalgebras can be found in [5]–[7].

The tensor product of a finite-dimensional Special Lie algebra and an exterior algebra as an associative algebra is a Lie superalgebra, which is called of Special type. This Lie superalgebra is actually isomorphic to a subalgebra of the finite-dimensional Lie superalgebra of Cartan type S. The main result of this paper is the complete determination for

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the superderivation algebras of the special Lie superalgebras, which says that the outer superderivations come from the outer derivations of the Lie algebras of Cartan type S. In particular, the first cohomology groups are determined.

2 Basics

Throughout this paper $\mathbb{F}$ is a field of characteristic $p > 3$. Fix two integers $m, n \geq 2$ and an m-tuple $\underline{t} := (t_1, t_2, \cdots, t_m)$. Let $\mathcal{O}(m,\ \underline{t})$ be the divided power algebra with $\mathbb{F}$-basis $\{x^{(\alpha)} \mid \alpha \in \mathbb{A}(m,\ \underline{t})\}$, where

$$\mathbb{A} := \mathbb{A}(m,\ \underline{t}) := \{\alpha \in \mathbb{N}_0^m \mid \alpha_i \leq \pi_i,\ i = 1, 2, \cdots, m\}, \qquad \pi_i := p^{t_i} - 1.$$

Let $W(m,\ \underline{t})$ be the generalized Witt algebra, i.e.,

$$W(m,\ \underline{t}) := \sum_{i=1}^{m} \mathcal{O}(m,\ \underline{t})\partial_i,$$

where ∂_i is the derivation of $\mathcal{O}(m,\ \underline{t})$ determined by

$$\partial_i(x^{(\varepsilon_j)}) = \delta_{ij}, \qquad i = 1, 2, \cdots, m.$$

Denote by $\Lambda(n)$ the $\mathbb{F}$-exterior superalgebra in n variables $x_{m+1}, \cdots, x_{m+n}$. Then

$$\mathfrak{W} := \Lambda(n) \otimes W(m,\ \underline{t})$$

is a Lie superalgebra with bracket:

$$[f \otimes D,\ g \otimes H] = fg \otimes [D,\ H], \qquad f, g \in \Lambda(n),\ D, H \in W(m,\ \underline{t}).$$

The natural $\mathbb{Z}$-grading of $\Lambda(n)$ and the standard $\mathbb{Z}$-grading of $W(m,\ \underline{t})$ induce a $\mathbb{Z}$-grading structure of $\mathfrak{W}$ with

$$\mathfrak{W}_i = \sum_{k+l=i} \Lambda(n)_k \otimes W(m,\ \underline{t})_l.$$

Note that $\mathfrak{W}$ is isomorphic to a subalgebra of the generalized Witt superalgebra (see [4]). For simplicity we write fD for

$$f \otimes D, \qquad f \in \Lambda(n),\ D \in W(m,\ \underline{t}).$$

Put

$$\mathrm{I}_0 := \{1, 2, \cdots, m\}, \qquad \mathrm{I}_1 := \{m+1, \cdots, m+n\}$$

and

$$\mathrm{I} := \mathrm{I}_0 \cup \mathrm{I}_1.$$

Let

$$\mathbb{B} := \{\langle i_1, i_2, \cdots, i_k\rangle \mid m+1 \leq i_1 < i_2 < \cdots < i_k \leq m+n\}.$$

For $u = \langle i_1,\ i_2,\ \cdots,\ i_k\rangle \in \mathbb{B}$, write

$$|u| := k, \qquad x^u := x_{i_1} x_{i_2} \cdots x_{i_k}, \qquad |\emptyset| = 0, \qquad x^{\emptyset} = 1.$$

For $i, j \in \mathrm{I}_0$, define

$$\partial_{ij} : \Lambda(n) \otimes \mathcal{O}(m,\ \underline{t}) \longrightarrow \Lambda(n) \otimes W(m,\ \underline{t})$$

so that for $f \in \Lambda(n) \otimes \mathcal{O}(m,\ \underline{t})$,

$$\partial_{ij}(f) = \partial_j(f)\partial_i - \partial_i(f)\partial_j.$$