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An Algorithm for Reducibility of 3-arrangements*

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Abstract: We consider a central hyperplane arrangement in a three-dimensional vector space. The definition of characteristic form to a hyperplane arrangement is given and we could make use of characteristic form to judge the reducibility of this arrangement. In addition, the relationship between the reducibility and freeness of a hyperplane arrangement is given.

Key words: hyperplane arrangement, reducibility, freeness

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1 Introduction

In this paper, we study the reducibility and freeness of central arrangements in a three-dimensional vector space. In [1] and [2], the authors gave the necessary and sufficient conditions of the reducibility of an arrangement $\mathcal{A}$. They connected the reducibility with one degree-branch of $D(\mathcal{A})$ and Beta invariant $\beta(\mathcal{A})$ (see [3]) respectively. However, it is needed a great quantity of calculation to judge the reducibility of $\mathcal{A}$. We give a simple method to judge the reducibility of an arrangement $\mathcal{A}$, which needs only the characteristic form of $\mathcal{A}$.

The freeness is an important property of an arrangement. Recently, there are many papers studying the freeness of arrangements. For example, Ziegler[4] and Yuzvinsky[5] gave some necessary and sufficient conditions of freeness. In addition, some conclusions on freeness of arrangements in a vector space of dimension three or higher were given in [6] and [7]. We study the freeness of arrangements from another angle of view, and the relationship between reducibility and freeness of a hyperplane arrangement is given in this paper.

The notions and symbols in this paper are the same as in [8] and [9].

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2 Basic Notions

Let $\mathbb{K}$ be a field and V be a vector space of dimension l on $\mathbb{K}$. A hyperplane H in V is an affine subspace of dimension $(l-1)$. A hyperplane arrangement $\mathcal{A}$ is a finite set of hyperplanes in V. If $\mathcal{A}$ consists of n hyperplanes, we write that

$$|\mathcal{A}| = n.$$

We call polynomial

$$Q(\mathcal{A}) = \prod_{H\in\mathcal{A}} \alpha_H \qquad (\ker \alpha_H = H)$$

the defining polynomial of $\mathcal{A}$. If

$$\bigcap_{H\in\mathcal{A}} H \neq \emptyset,$$

then we call $\mathcal{A}$ central, otherwise, we call $\mathcal{A}$ non-central. The dimension of $\mathcal{A}$, $\dim \mathcal{A}$, is defined to be

$$\dim \mathcal{A} = l.$$

Let W be the space spanned by the normals to the hyperplanes in $\mathcal{A}$, and the rank of $\mathcal{A}$, $\mathrm{rank}(\mathcal{A})$, be the dimension of W. We say that $\mathcal{A}$ is essential if

$$\mathrm{rank}(\mathcal{A}) = \dim(\mathcal{A}).$$

Let

$$L = L(\mathcal{A})$$

be the set of nonempty intersections of elements of $\mathcal{A}$.

Let $(\mathcal{A}_1,\ V_1)$ and $(\mathcal{A}_2,\ V_2)$ be two arrangements and

$$V = V_1 \oplus V_2.$$

Define the product arrangement $(\mathcal{A}_1 \times \mathcal{A}_2,\ V)$ by

$$\mathcal{A}_1 \times \mathcal{A}_2 = \{H_1 \oplus V_2 \mid H_1 \in \mathcal{A}_1\} \cup \{V_1 \oplus H_2 \mid H_2 \in \mathcal{A}_2\}.$$

Call the arrangement $(\mathcal{A},\ V)$ reducible if after a change of coordinates, we get

$$\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2,$$

otherwise, call $\mathcal{A}$ irreducible.

Let

$$S = S(V^*)$$

be the symmetric algebra of the dual space V^* of V. If $x_1, \cdots, x_l$ is a basis for V^*, then

$$S \simeq \mathbb{K}[x_1,\ \cdots,\ x_l].$$

Let

$$\mathrm{Der}_{\mathbb{K}}(S) = \{\theta : S \to S \mid \theta(fg) = f\theta(g) + g\theta(f),\ f, g \in S\}.$$

Let S_p denote the $\mathbb{K}$-vector subspace of S consisting of 0 and the homogeneous polynomials of degree p for $p \geq 0$. A nonzero element $\theta \in \mathrm{Der}_{\mathbb{K}}(S)$ is homogeneous of polynomial degree p if

$$\theta = \sum_{k=1}^{l} f_k \frac{\partial}{\partial x_k} \in \mathrm{Der}_{\mathbb{K}}(S),$$