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Three-stage Stiffly Accurate Runge-Kutta Methods for Stiff Stochastic Differential Equations*

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Abstract: In this paper we discuss diagonally implicit and semi-implicit methods based on the three-stage stiffly accurate Runge-Kutta methods for solving Stratonovich stochastic differential equations (SDEs). Two methods, a three-stage stiffly accurate semi-implicit (SASI3) method and a three-stage stiffly accurate diagonally implicit (SADI3) method, are constructed in this paper. In particular, the truncated random variable is used in the implicit method. The stability properties and numerical results show the effectiveness of these methods in the pathwise approximation of stiff SDEs.

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1 Introduction

In this paper we consider numerical methods for the strong solution of the stochastic differential equations

$$\mathrm{d}\boldsymbol{y}(t) = f(\boldsymbol{y}(t))\mathrm{d}t + g(\boldsymbol{y}(t)) \circ \mathrm{d}W(t), \qquad \boldsymbol{y}(t_0) = \boldsymbol{y}_0,\ t \in [t_0, T],\ \boldsymbol{y} \in \mathbf{R}^m \tag{1.1}$$

in Stratonovich form, which can be written in autonomous form without loss of generality, where $W(t)$ is a Wiener process, whose increment

$$\Delta W(t) = W(t + \Delta t) - W(t)$$

is a Gaussian random variable $N(0, \Delta t)$.

For solving SDEs, Rümelin[1] has presented a general class of stochastic Runge-Kutta

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(SRK) methods with strong order 1.0 given by

$$\boldsymbol{Y}_i = \boldsymbol{y}_n + h\sum_{j=1}^{s} a_{ij} f(\boldsymbol{Y}_j) + J_1 \sum_{j=1}^{s} b_{ij} g(\boldsymbol{Y}_j), \qquad i = 1, \cdots, s,$$

$$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\sum_{j=1}^{s} \alpha_j f(\boldsymbol{Y}_j) + J_1 \sum_{j=1}^{s} \gamma_j g(\boldsymbol{Y}_j),$$

which can be represented by the so-called Butcher tableau

$$\begin{array}{c|cc} & \boldsymbol{A} & \boldsymbol{B} \\ \hline & \boldsymbol{\alpha}^\top & \boldsymbol{\gamma}^\top \end{array}$$

Here $\boldsymbol{A} = (a_{ij})$ and $\boldsymbol{B} = (b_{ij})$ are $s \times s$ matrices of real elements, while $\boldsymbol{\alpha}^\top = (\alpha_1, \cdots, \alpha_s)$ and $\boldsymbol{\gamma}^\top = (\gamma_1, \cdots, \gamma_s)$ are row vectors in $\mathbf{R}^s$, h is a step size, and $J_1 = \int_{t_n}^{t_{n+1}} \circ \mathrm{d}W$.

In determinate case, the implicit methods are suitable for solving stiff ordinary differential equations (see [2]). However, there is no simple stochastic counterpart of the deterministic implicit Euler method, i.e., the method

$$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + f(\boldsymbol{y}_{n+1})h + g(\boldsymbol{y}_{n+1})\Delta W_n$$

fails because, for example, for linear Itô equation

$$\mathrm{d}y(t) = ay(t)\mathrm{d}t + by(t)\mathrm{d}W(t), \qquad y(t_0) = y_0, \ y \in \mathbf{R}^1,$$

we have

$$E|(1 - ah - b\Delta W_n)^{-1}| = +\infty.$$

In this paper we propose to make use of the truncated random variable and stiff accuracy to avoid analogous case.

In recent years many efficient numerical methods have been constructed for solving different types of SDEs with different properties (for example, see [3]–[8]). In particular, several authors have presented the different efficient implicit methods for stiff SDEs (see [9]–[13]). Milstein *et al.*[11] have introduced the balanced implicit methods. Their strong convergence order is merely 0.5 all right, but the results show effectiveness of these methods in the pathwise approximation of stiff SDEs. Burrage and Tian[9] presented a class of stiffly accurate methods based on the two-stage Runge-Kutta methods and splitting techniques for stiff SDEs. Wang[14] has considered three-stage explicit and semi-implicit methods.

In this paper we are to construct a class of stiffly accurate methods based on the three-stage diagonally implicit Runge-Kutta methods for stiff SDEs. In Section 2 we present a diagonally implicit method and a semi-implicit method which are based on a classical deterministic three-stage Runge-Kutta method. The stability properties and numerical results of these methods are reported in Sections 3 and 4, respectively.

2 Stiffly Accurate Diagonally Implicit SRK Methods

For computing solutions of SDE (1.1), we present a class of three-stage stiffly accurate diagonally implicit SRK methods, namely,