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Boundedness of Some Singular Integral Operators on Morrey-Herz Spaces over Locally Compact Vilenkin Groups*

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Abstract: In this paper the boundedness results for some fractional and singular integral operators on the homogeneous Morrey-Herz spaces over locally compact Vilenkin groups are shown.

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1 Introduction

On the Euclidean space $\mathbf{R}^n$, many important results for sublinear operators are obtained. For example, Li and Yang[1] established the boundedness of some sublinear operators on Herz-type spaces in 1996. And Lu and Xu[2] further studied the boundedness for sublinear operators on Morray-Herz spaces in 2005.

However, some researches revealed that many classical results for sublinear operators on $\mathbf{R}^n$ still hold on locally compact Vilenkin groups G. In 1998, Yang[3] replaced $\mathbf{R}^n$ by a locally compact Vilenkin groups G and investigated the $L^p(G)$-boundedness of sublinear operators with weaker conditions, which implies its $L^p_{|x|^\alpha}(G)$-boundedness. And Lu and Yang[4] established the boundedness of some sublinear operators in weighted Herz spaces over G. In 2001, Kitada and Yang[5] studied the boundedness of potential operators and the maximal operators associated with them in the weighted Herz space over G. And recently,

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Wu[6] considered the boundedness of commutators on Morrey-Herz spaces $M\dot{K}^{\alpha,\lambda}_{p,q}(G)$. Motivated by [1]–[6], the main purpose of this paper is to further generalize the relative results for theirs and establish some boundedness results of fractional and singular integral operators on $M\dot{K}^{\alpha,\lambda}_{p,q}(G)$. Before stating our main results, we first give some basic concepts and definitions.

Throughout this paper, we denote by G a locally compact Abelian group containing a strictly decreasing sequence of compact open subgroups $\{G_n\}_{n=-\infty}^{\infty}$ such that

(i) $\bigcup\limits_{n=-\infty}^{\infty} G_n = G$ and $\bigcap\limits_{n=-\infty}^{\infty} G_n = \{0\}$;

(ii) $\sup\{\text{order}(G_n/G_{n+1}) : n \in \mathbf{Z}\} = B < \infty$.

Let $\mathit{\Gamma}$ denote the dual group of G, and $\mathit{\Gamma}_n$ denote the annihilator of G_n for each $n \in \mathbf{Z}$. That is,

$$\mathit{\Gamma}_n = \{\gamma \in \mathit{\Gamma} : \gamma(x) = 1, \ \forall x \in G_n\}.$$

Then $\{\mathit{\Gamma}_n\}_{n=-\infty}^{\infty}$ is a strictly increasing sequence of compact open subgroups of $\mathit{\Gamma}$ such that

(i)$'$ $\bigcup\limits_{n=-\infty}^{\infty} \mathit{\Gamma}_n = \mathit{\Gamma}$ and $\bigcap\limits_{n=-\infty}^{\infty} \mathit{\Gamma}_n = \{1\}$;

(ii)$'$ $\text{order}(\mathit{\Gamma}_n/\mathit{\Gamma}_{n+1}) = \text{order}(G_n/G_{n+1})$.

We choose Haar measure $\mathrm{d}\mu$ (or $\mathrm{d}x$) on G and $\mathrm{d}\gamma$ on $\mathit{\Gamma}$ so that

$$|G_0| = |\mathit{\Gamma}_0| = 1,$$

where $|A|$ denotes the Haar measure of a measurable subset A of G, or $\mathit{\Gamma}$. Then

$$|G_n|^{-1} = |\mathit{\Gamma}_n| \equiv m_n, \qquad n \in \mathbf{Z}.$$

Since $2m_n \le m_{n+1} \le Bm_n$, $n \in \mathbf{Z}$, it follows that for any $a > 0$, $k \in \mathbf{Z}$,

$$\sum_{n=k}^{\infty} (m_n)^{-a} \le C(m_k)^{-a}, \qquad \sum_{n=-\infty}^{k} (m_n)^{a} \le C(m_k)^{a}.$$

If we define the function $d : G \times G \to \mathbf{R}$ by

$$d(x, y) = \begin{cases} 0, & x - y = 0; \\ (m_n)^{-1}, & x - y \in G_n \backslash G_{n+1}, \end{cases}$$

then d defines a metric on $G \times G$ and the topology on G induced by this metric is the same as the original topology on G. For $x \in G$, set

$$|x| = d(x, \ 0).$$

Then

$$|x| = (m_n)^{-1} \qquad \text{iff } x \in G_n \backslash G_{n+1}.$$

If $I = x + G_n$, we say I is a coset of G, where $x \in G$ and $n \in \mathbf{Z}$.

Examples of these groups are given in §4.1.2 of [7]. An additional example is the additive group of a local field, see [8] for details. For more details, we refer to [9]–[12].

Here and in what follows, for any non-negative weight function ω, any measurable function f on G and any $q \in (0, \infty]$, we denote by $L^q_\omega(G)$ the Lebesgue space on G with respect to the weight measure $\omega(x)\mathrm{d}x$. And we write

$$\|f\|_{L^q_\omega(G)} = \Big(\int_G |f(x)|^q \omega(x) \mathrm{d}x \Big)^{\frac{1}{q}}$$

with the usual modification when $q = \infty$. If $\omega \equiv 1$, we denote $L^q_\omega(G)$ simply by $L^q(G)$.