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On Complete Convergence for Arrays of Rowwise Strong Mixing Random Variables*

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Abstract: In this paper, we present a general method to prove the complete convergence for arrays of rowwise strong mixing random variables, and give some results on complete convergence under some suitable conditions. Some Marcinkiewicz-Zygmund type strong laws of large numbers are also obtained.

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1 Introduction

Let $\{\Omega,\ \mathcal{F},\ P\}$ be a probability space, and $\{X_n : n \geq 1\}$ be a sequence of random variables defined on this space.

Definition 1.1 *The sequence $\{X_n : n \geq 1\}$ is said to be α-mixing or strong mixing if*

$$\alpha(n) = \sup_{m\geq 1}\{|P(AB) - P(A)P(B)| : A \in \mathcal{F}_{-\infty}^{m},\ B \in \mathcal{F}_{m+n}^{\infty}\} \to 0 \qquad \text{as } n \to \infty,$$

where $\mathcal{F}_m^n$ denotes the σ-field generated by $\{X_i : m \leq i \leq n\}$.

The strong mixing coefficient was introduced by Rosenblatt[1] and have been commonly employed in establishing limiting results for time series and random fields (see [2]–[4]). Recently, Genon-Gatahot *et al.*[5] showed that continuous time diffusion models and stochastic volatility models were strong mixing. Strong mixing random variables, because of their wide

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applications, have been studied in many different aspects: the moment inequalities (see [6]–[7]), the center limit theorem (see [8]), the strong approximation theorem (see [9]), and the complete convergence (see [10]–[12]).

For α-mixing sequences, Hipp[10] presented the following result.

Theorem 1.1 *Let $1/2 < \alpha \le 1$, $2 < r \le \infty$, $1/\alpha < p < r$, and $\{X_n : n \ge 1\}$ be a strictly stationary α-mixing sequence of random variables with $EX_1 = 0$ and $(E|X_1|^r)^{1/r} < \infty$. Assume that $\sum_{n=1}^{\infty} \alpha^{1/\theta}(n) < \infty$ for some $\theta > [2 + r/(r-p)]p\alpha/(p\alpha - 1)$. Then*

$$\sum_{n=1}^{\infty} n^{p\alpha-2} P\left\{ \max_{1\le i\le n} \left\{ \left| \sum_{j=1}^{i} X_j \right| \right\} \ge \varepsilon n^{\alpha} \right\} < \infty, \qquad \varepsilon > 0.$$

However, a contrary example to Hipp's conclusion was given by Berbee[13] when $r = \infty$, i.e., in the case of $|X_1|$ bounded. Shao[11] also showed that Theorem 1.1 is quite possibly not true when $r < \infty$. In this paper, we present a general method to prove the complete convergence for arrays of rowwise strong mixing random variables, and give some results on complete convergence under some suitable conditions. Some Marcinkiewicz-Zygmund type strong laws of large numbers are also obtained.

Now, we give two definitions needed in the further part of the paper.

Definition 1.2 *An array $\{X_{ni} : i \ge 1,\ n \ge 1\}$ of random variables is said to be stochastically dominated by a random variable X if there exists a constant D such that*

$$P\{|X_{ni}| > x\} \le DP\{|X| > x\}, \qquad x \ge 0,\ i \ge 1,\ n \ge 1.$$

Definition 1.3 *A real-valued function $l(x)$, positive and measurable on $[A, \infty)$ for some $A > 0$, is said to be slowly varying if*

$$\lim_{x\to\infty} \frac{l(\lambda x)}{l(x)} = 1, \qquad \lambda > 0.$$

Throughout the sequel, C represents a positive constant although its value may change from one appearance to the next; $[x]$ indicates the maximum integer no larger than x; $I[B]$ denotes the indicator function of the set B and $\|X\|_q = (E|X|^q)^{1/q}$.

2 Main Results

The following lemmas are useful in our study.

Lemma 2.1[7] *Let $q > 2$, $\delta > 0$, and $\{X_n : n \ge 1\}$ be an α-mixing sequence of random variable with $EX_i = 0$ and $E|X_i|^{q+\delta} < \infty$. Suppose that $\theta > q(q+\delta)/(2\delta)$ and $\alpha(n) \le Cn^{-\theta}$ for some $C > 0$. Then, for any $\epsilon > 0$, there exists a positive constant $K = K(\epsilon, q, \delta, \theta, C) < \infty$ such that*

$$E \max_{1\le j\le n} \left\{ \left| \sum_{i=1}^{j} X_j \right|^q \right\} \le K\left\{ n^{\epsilon} \sum_{i=1}^{n} E|X_i|^q + \left(\sum_{i=1}^{n} \|X_i\|_{q+\delta}^2 \right)^{q/2} \right\}.$$