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Some Characterizations of Left Weakly Regular Ordered Semigroups*

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Abstract: In this paper, the notion of left weakly regular ordered semigroups is introduced. Furthermore, left weakly regular ordered semigroups are characterized by the properties of their left ideals, right ideals and (generalized) bi-ideals, and also by the properties of their fuzzy left ideals, fuzzy right ideals and fuzzy (generalized) bi-ideals.

Key words: left weakly regular ordered semigroup, generalized bi-ideal, fuzzy left ideal, fuzzy right ideal, fuzzy bi-ideal, fuzzy generalized bi-ideal

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1 Introduction

Let S be a nonempty set. A fuzzy subset of S is, by definition, an arbitrary mapping $f : S \longrightarrow [0, 1]$, where $[0, 1]$ is the usual interval of real numbers. The important concept of a fuzzy set put forth by Zadeh[1] in 1965 has opened up keen insights and applications in a wide range of scientific fields. Recently, a theory of fuzzy sets on ordered semigroups have been developed (see [2]–[6]). Following the terminology given by Zadeh, if S is an ordered semigroup, fuzzy sets in ordered semigroups S have been first considered by Kehayopulu and Tsingelis[2], who then defined fuzzy analogous for several notations that have been proved to be useful in the theory of ordered semigroups. Moreover, each ordered groupoid can be embedded into an ordered groupoid having a greatest element (poe-groupoid) in terms of fuzzy sets (see [3]). The concept of ordered fuzzy points of an ordered semigroup S has been

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first introduced by Xie and Tang[7], and prime fuzzy ideals of an ordered semigroup S were studied in [8].

The regularities were first appeared in rings and semirings (see [9]). As we know, regular rings play an important role in the abstract algebra. Several kinds of regularities have been discussed universally, such as strongly regular, completely regular, weakly regular and left (right) regular. They are all equivalent in a commutative ring or semiring. Brown and McCoy[10] considered the notion of weakly regular rings. These rings were later studied by Ramamurthy[11] and others (see [12]). Adopting this notion to semigroup Ahsan *et al.*[13] considered weakly regular semigroups. Recently, Kehayopulu extended those similar fuzzy results to ordered semigroups (see [5], [6]). In [14], the authors characterized regular ordered semigroups and intra-regular ordered semigroups in terms of fuzzy left ideals, fuzzy right ideals, fuzzy (generalized) bi-ideals and fuzzy quasi-ideals. As a continuation of the study undertaken by Kehayopulu and Tsingelis[5],[6] and the authors (see [14]), we first introduce the concept of left weakly regular ordered semigroups in the present paper. Then left weakly regular ordered semigroups are characterized by the properties of their left ideals, right ideals and (generalized) bi-ideals, and also by fuzzy left ideals, fuzzy right ideals and fuzzy (generalized) bi-ideals. As an application of the results of this paper, the corresponding results of unordered semigroups are also obtained.

2 Preliminaries and Some Notations

Throughout this paper S stands for an ordered semigroup unless stated otherwise. A function f from S to the real closed interval $[0, 1]$ is a fuzzy subset of S. The ordered semigroup S itself is a fuzzy subset of S such that

$$S(x) = 1, \qquad x \in S$$

(the fuzzy subset S is also denoted by 1, see [5]). Let f and g be two fuzzy subsets of S. Then the inclusion relation $f \subseteq g$ is defined by

$$f(x) \leq g(x), \qquad x \in S,$$

and $f \cap g$, $f \cup g$ are defined by

$$(f \cap g)(x) = \min\{f(x),\ g(x)\} = f(x) \wedge g(x),$$
$$(f \cup g)(x) = \max\{f(x),\ g(x)\} = f(x) \vee g(x)$$

for all $x \in S$, respectively. The set of all fuzzy subsets of S is denoted by $F(S)$. One can easily see that $(F(S), \subseteq, \cap, \cup)$ forms a complete lattice.

Let $(S,\ \cdot\ , \leq)$ be an ordered semigroup. For $x \in S$, we define

$$A_x := \{(y, z) \in S \times S \mid x \leq yz\}.$$

The product of $f \circ g$ is defined by

$$(f \circ g)(x) = \begin{cases} \bigvee\limits_{(y,z)\in A_x} [\min\{f(y),\ g(z)\}], & \text{if } A_x \neq \emptyset; \\ 0, & \text{if } A_x = \emptyset, \end{cases} \qquad x \in S.$$

It is well known (see Theorem of [2]) that this operation "$\circ$" is associative.